Simple circuits

Resistances in circuits

Analogy of water flow and electric current

An electrical circuit consists of a closed loop with a number of different elements through which electric current passes. This loop may be made up of a number of sub-loops. In order to understand the concepts of voltage, current, resistance, and conductance, we will use the following analogy.

Consider a water pump as presented in _Figure 1_ below, which lifts water through a height, H, up to reservoir A, from which the water flows down a pipe of diameter, D, to reservoir B below. The pump then takes the water from reservoir B and pumps it back up the same height, H, to reservoir A. This water circuit is a closed loop. The pump gives potential energy (gravitational) to the water in the same way that a battery in an electrical circuit provides energy to the electrical charges. The height difference (H) between A and B determines the magnitude of the water droplets' potential energy ($E = mgH$), which is released when they fall down the pipe. This is analogous to the voltage difference between the positive and negative terminals of a battery, or any other source of voltage. It determines the rate at which the electrical charges move (current).

The amount of droplets passing a given point in one second is analogous to the electrical current, which corresponds to the number of electrons (with a total charge measured in Coulombs) that pass a given point in one second. The unit of current is Amperes. The diameter of the water pipe can relate to the resistance of an electrical component. The larger the diameter of the pipe, the greater the flow rate of water is (the smaller the diameter, the smaller is the flow rate). Assuming the applied voltage is the same, thicker wires of the same material can carry more current. The resistance is the ability to restrict the current flow which is measured in Ohms (Ω).
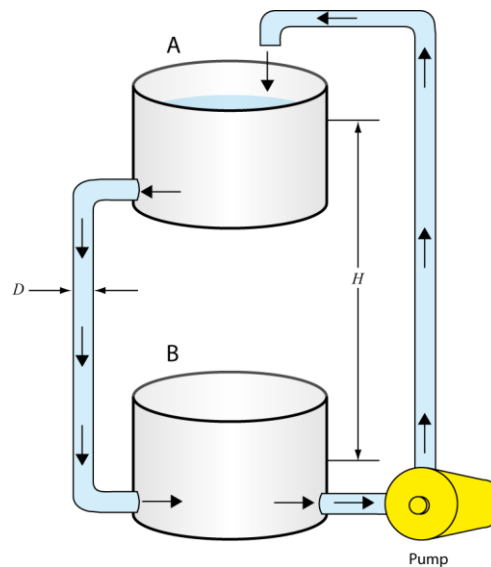

Figure 1 - Analogy of water flow and electric current for a resistor

Current convention

In the wires of electrical circuits, negatively-charged electrons carry the current. However, in other devices such as batteries, positively-charged ions may also contribute to the current. Positive charges move in the opposite direction to that of the electrons. Historically, the direction of electrical current is taken to be that of positive charges.

Ohm's law

Ohm's law describes the relationship between electric potential, current, and resistance. In this experiment, you will examine how combinations of voltages and resistors affect both the energy and the flow rate of charge in electrical circuits. Ohm's law states that the voltage difference, ΔV, across two points of an element is directly proportional to the current, I, going through the element:

$$\Delta V = RI, \tag{eq. 1}$$

with R being the resistance of the element as discussed above. Ohm's law implies that a plot of voltage as a function of current in a circuit can be used to determine the resistance (the slope of the straight line graph equals the resistance). The resistance value of commercial resistor can be obtained from colored bars printed on it (see the _Tutorial - Using a multimeter_) or measured using an Ohmmeter. For combinations of more than one resistor, methods for calculating the total resistance will be investigated during this experiment.

Kirchhoff's rules

We can analyse a simple circuit using Ohm's law and the rules for series and parallel combinations of resistors. For more complex circuits we use Kirchhoff's rules. These two rules allow one to set up sets of equations which can be algebraically manipulated to solve for the unknown quantities (usually the current through all parts of a circuit).

The Junction Rule (Conservation of Charge):
The sum of the currents entering any junction must equal the sum of the currents leaving that junction. A junction is any point in a circuit where the current is split or is re-joined. This first rule is a statement of conservation of charge. Whatever current enters a given point in a circuit must leave that point because charge cannot build or disappear.

The Loop Rule (Conservation of Energy):
The sum of the potential energy differences (voltage changes) across each element around any closed circuit loop must be zero.

The initial step in applying Kirchhoff's rules is to choose the direction of flow for all the currents of the circuit. In the example shown in _Figure 2_, we can be certain of these directions of flow. In more complex circuits the directions must be "guessed" and if a final result leads to a negative current, then the magnitude is still correct but we know the direction must be changed. If we apply the junction rule to the example circuit shown, we find at the junction labelled by point "c" we get;

$$I_1 = I_2 + I_3. \tag{eq. 2}$$

We know that at point "d" the two currents re-join and should be equal to I_1. Applying the junction rule at point "d" would just give the same equation and thus no new information. The Loop Rule will be needed to continue. The second rule is equivalent to the law of conservation of energy. Any charge that moves around a closed loop in a circuit, on returning to its original point will have the same potential energy or V (voltage) with which it started. It must gain as much energy as it loses.
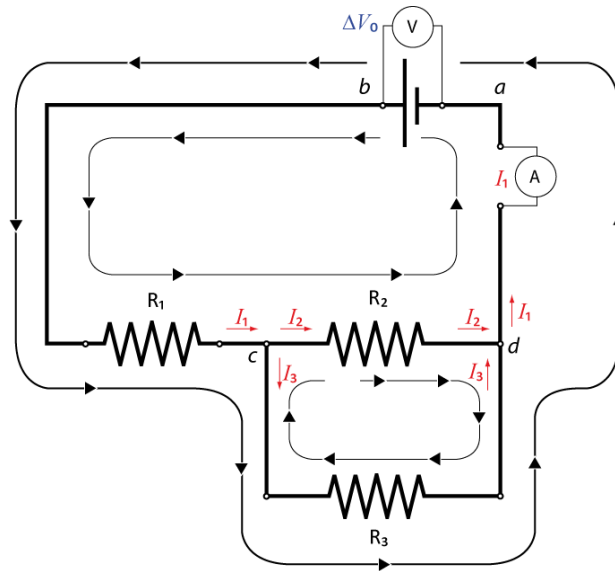
Figure 2 – Currents in a mixed circuit to analyze it using Kirchhoff's rules

A charge's energy may decrease across a resistor by $\Delta V = -IR$ or increase by flowing through a power source such as a battery from negative (−) to positive (+). However, when the charge reaches the starting point of a loop, its energy will be back to its original level.

In the circuit shown in *Figure 2*, there are three possible loops that could be used to build conservation of energy equations. However since we have three unknowns to find (I_1, I_2, I_3) and already have one independent equation (from the Junction Rule, $I_1 = I_2 + I_3$), we only need two additional loops to get two more equations. (i.e. with three independent equations one can algebraically solve for three unknowns). Thus two loops are chosen as indicated in the *Figure 2* (the loops are shown inside the circuit).

For the larger loop, one starts at point "*a*" at zero potential and travels through the power source from negative (−) to positive (+), giving a voltage rise of ΔV_0. Passing through the first resistor gives a voltage drop that depends on I_1 namely, $\Delta V = -R_1 I_1$. In this loop the next resistor to go through gives the final voltage drop of $\Delta V = -R_2 I_2$ and then we return back to point "*a*". Thus the second equation is:

$$\Delta V_0 - R_1 I_1 - R_2 I_2 = 0. \tag{eq. 3}$$

The second loop chosen has no source of potential but if we examine the behaviour of a test charge forced around this loop, its potential will drop when passing through a resistor in the same direction as the current OR will rise if going the opposite direction of the current (like going up or down a waterfall). Thus if we pick point "*c*" as the starting point then we are traveling with the current through resistor R_2 (voltage drops) and go against the current through resistor R_3 (voltage rises) and we return back to "*c*". This gives the equation:

$$-R_2 I_2 + R_3 I_3 = 0. \tag{eq. 4}$$

We now have three equations (eqs. 2, 3, and 4) and three unknowns (I_1, I_2, I_3). You can now choose any way to solve the three equations and determine the unknown currents.

Capacitors in circuits

Analogy to the capacitor in electrical circuit

An example of a capacitor (shown in *Figure 3*), is a hollow sphere which is divided into two equal volumes, A and B, with a flexible rubber circular sheet separating them. The sphere is filled with water and it has two valves. The two valves are connected to a pump. When you turn on the pump, it pulls water from division A and pushes it into division B. As a result of the work done on the water, the rubber sheet is distorted and an elastic energy is stored in it. Divisions A and B are analogous to the two plates of the capacitor. The pump is analogous to the battery. The rubber sheet is analogous to the dielectric material between the plates. The elastic energy stored in the rubber sheet is analogous to electrical energy stored in the capacitor. It is important to mention that we did not add water to the sphere; we transfer water from A to B. The result is storing elastic energy. Similarly, we do not add charges to the capacitor when we "charge" it, we simply transfer charges from one plate to the other. The result is storing electrical energy in the capacitor.
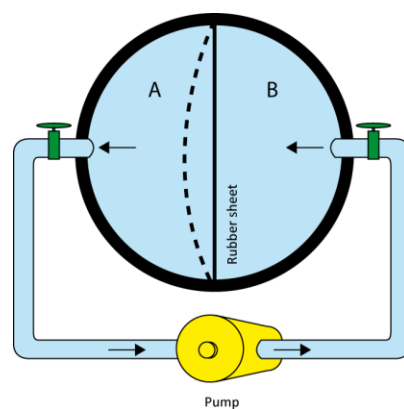

Figure 3 - Analogy of water and electric current for a capacitor

RC circuit (resistor-capacitor)

In this experiment we investigate the behaviour of circuits containing a resistor and a capacitor in series. The rate of decrease of charge in the capacitor at any instant is proportional to the charge remaining at that time. The only function having the property that its rate of change (i.e., its derivative) is proportional to the function itself is the exponential function. If the initial charge is Q_0, then we can express the instantaneous charge in the capacitor using an exponential function:

$$Q = Q_0 e^{-t/RC} \, , \tag{eq. 5}$$

where t is the time, R is the resistance of the resistor, and C is the capacitance of the capacitor (measured in Farads). The product RC is called the time constant, or relaxation time of the circuit. As equation (5) shows, after a time t_0 equal to RC, the charge has dropped to a fraction $Q/Q_0 = e^{-1} = 0.368 = 36.8\%$ of its original value. The charge continues to drop by this ratio over any subsequent interval of time t_0. Finally, since $Q = CV$, the voltage difference between the capacitor plates in a RC circuit will also be an exponential function with the same time constant equal to RC.

Suggested reading

Students taking	Suggested reading	
PHY 1122	Section 24.2, Chapters 25 and 26	Young, H. D., Freedman, R. A., *University Physics with Modern Physics, 13th edition*. Addison-Wesley (2012).
PHY 1322	Section 26.3, Chapters 27 and 28	Serway, R. A., Jewett, J. W., *Physics for Scientists and Engineers with Modern Physics, eight edition*. Brooks/Cole (2010).
PHY 1124	Section 25.4, Chapters 26 and 27	Halliday, D., Resnick, R., Walker, J., *Fundamentals of Physics, 9th edition*. Wiley (2011).

Objectives

Part 1 – Measuring a resistance value

- ✓ Read the colour coded value of a resistor.
- ✓ Use an Ohmmeter.

Part 2 – Ohm's law

- ✓ Prepare circuits on a solderless breadboard.
- ✓ Use a voltmeter and an ammeter.
- ✓ Investigate all variables involved in the Ohm's law relationship.

Part 3 – Combination of resistors

- ✓ Investigate simple electrical circuits: series, parallel, and mixed.
- ✓ Understand the effective resistance concept.

Part 4 – Voltages and currents in circuits (Kirchhoff's rules)

- ✓ Experimentally verify the Kirchhoff's rules of circuit analysis for a simple example.

Part 5 – Combinations of capacitors & RC circuits

- ✓ Understand the effective capacitor concept.
- ✓ Observe how capacitors behave in RC circuits.
- ✓ Familiarize yourself with the use of an oscilloscope.

Materials

- Computer equipped with *Logger Pro*
- Computer equipped with the National Instrument *myDAQ* virtual instruments
- National Instrument *myDAQ* data acquisition system
- Wire kit and breadboard
- Resistors (470 Ω, 1 kΩ and 3.3 kΩ) and capacitors (0.1 μF and 0.22 μF)

Safety warnings

You should always disconnect your circuit from the power source to use the Ohmmeter. You should also always double check your circuit before adding the power source. In case of doubt, ask your TA to verify your circuit.

References for this manual

- Dukerich, L., *Advanced Physics with Vernier – Beyond mechanics*. Vernier software and Technology (2012).
- *Basic Electricity*. PASCO scientific (1990).
- User guide and specifications for the NI myDAQ

Procedure

Part 1 – Measuring a resistance value

Step 1. Based on the colour codes in *Table 1*, identify the three resistors R_1, R_2 and R_3.

Step 2. Using the resistor colour chart in the *Tutorial - Using a multimeter*, identify the resistance values of the three resistors and complete columns 3, 4 and 5 of *Table 1*.

Step 3. Using the Fluke multimeter (the yellow one), measure the resistance values of the three resistors.
- Select the Ω symbol.
- Connect the black and red cable with alligator clips to both branches of one resistor.
- Record the resistance value and complete columns 6 and 7 of *Table 1*. Refer to the *Tutorial - Using a multimeter* to complete column 7.

Step 4. Compare the colour coded value with your measurement and complete the last column of *Table 1*.

Part 2 – Ohm's law

Step 1. Turn on your computer and launch the **Digital Multimeter** and the **DC Level** (power supply) programs (these programs should be available in the NIElvis folder from the computer's desktop).

Step 2. Assemble the circuit below using your 1 kΩ resistor, the Fluke multimeter as a voltmeter and the myDAQ multimeter as an ammeter. The power supply is the red and black wires labelled **AO0** and **AGND** coming out of your myDAQ unit. Refer to the tutorials in order to make the proper connections. Optional: Ask your TA to verify your circuit before going to the next step.
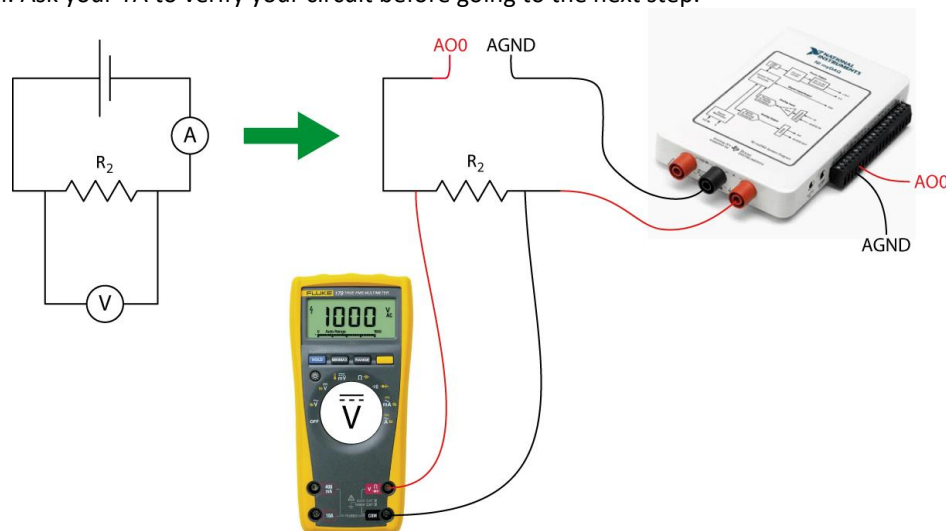
Step 3. Select the DC current ammeter in the myDAQ multimeter window (third button in the **Measurement Settings** section). Select **Auto** in the **Mode** menu. Click **Run** to start making measurements.

Step 4. Set the **Voltage Level** of the power supply to 0.25 V. Make sure the **Channel Settings** is set to **myDAQ1/ao0**. Click **Start** to turn the power on. Read the voltage and the current using your multimeters. Enter the values in *Table 2*.

Step 5. Increase the voltage to by 0.25 V. Repeat the measurements and complete *Table 2*. Click **STOP** to turn off the power supply.

Step 6. Launch the Logger Pro program. Prepare a graph of the Voltage at the resistor (in V) vs. Current (in A). This is your Graph 1. Arrange your graph to get a proper display according to the tutorial *How to prepare a graph*.

Step 7. Perform a linear fit of Graph 1. Select the graph, click **Analyze** then **Linear Fit**.

Step 8. Save your Graph 1.
- Click **File** then **Page Setup…** and select the **Landscape** orientation. Click **OK**.
- Select **File** then **Print Graph….** When the printing options windows opens, add your name and the one of your partner(s) in the field **Name:**. Click **OK**.
- Make sure to select the **CutePDF** as a printer and click **OK** again.
- Save your graph on the computer. You will need to print this graph before submitting your report, so you should either send the file to yourself by email or transfer it to a USB key.

Step 9. We strongly recommend that you save all the work you do during the lab in case you need to review it later. Click **File/Save As…** to save your experiment file (suggested name: *Ohm_YOUR_NAMES.cmbl*). You can either send the file to yourself by email or save it on a USB key.

Part 3 – Combination of resistors

Step 1. Using the ohmmeter, measure the effective resistance of various combinations of resistors in series to complete *Table 3*. Assemble your resistors in series using the breadboard.
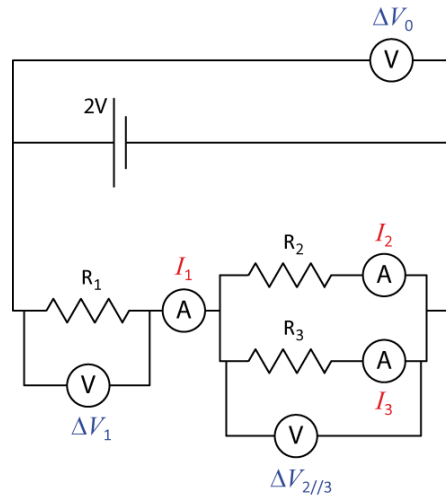
Step 2. Using the ohmmeter, measure the effective resistance of various combinations of resistors in parallel to complete *Table 4*. Assemble your resistors in parallel using the breadboard.

Step 3. Using the ohmmeter, measure the effective resistance of resistor 1 in series with resistors 2 and 3 in parallel (see below). Assemble this mixed circuit using the breadboard.
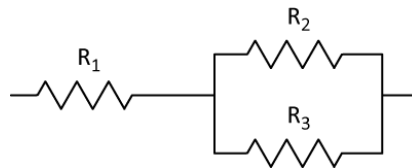
Part 4 – Voltages and currents in circuits (Kirchhoff's rules)

Step 1. Add a power supply to the mixed circuit you just worked with. Set the power supply to 2 V, turn it on and measure the voltage drops (ΔV_0, ΔV_1 and $\Delta V_{2//3}$) and the currents (I_1, I_2 and I_3) going through each branch of the circuit as labelled below. As before, use the Fluke voltmeter for your voltage measurements and the myDAQ ammeter to measure your currents. Turn off the power supply when you are done.


Part 5 – Combinations of capacitors & RC circuits

Step 1. Using the Fluke multimeter, measure the capacitance values of the two capacitors provided (C_1: 0.1 µF and C_2: 0.22 µF). Enter the values in *Table 5*.

Step 2. Measure the effective capacitance of these two capacitors connected in series or in parallel. Assemble your capacitors using the breadboard and complete *Table 5*.

Step 3. Make sure the **DC Level** power supply and the multimeter windows are turned off on your computer. Launch the **Function generator** and the **Oscilloscope** programs (these programs should be available from the NIElvis folder on the desktop).

Step 4. Assemble the RC circuit below using your 1 kΩ resistor and the 0.22 µF capacitor.
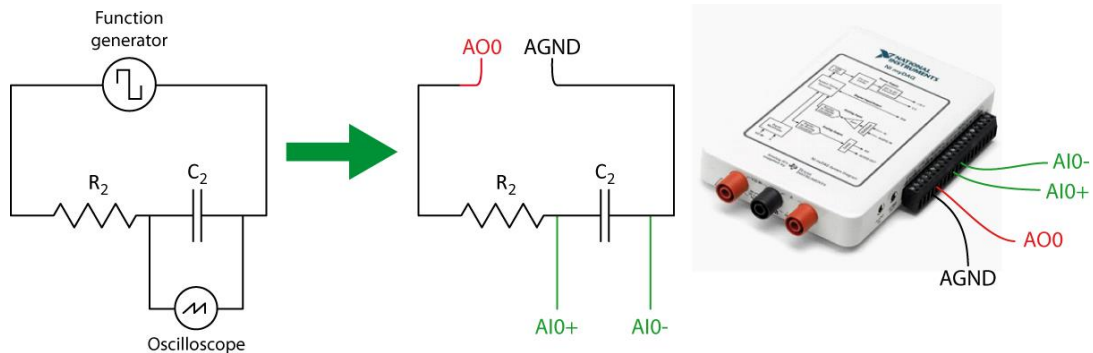
Step 5. Set the function generator as follow:
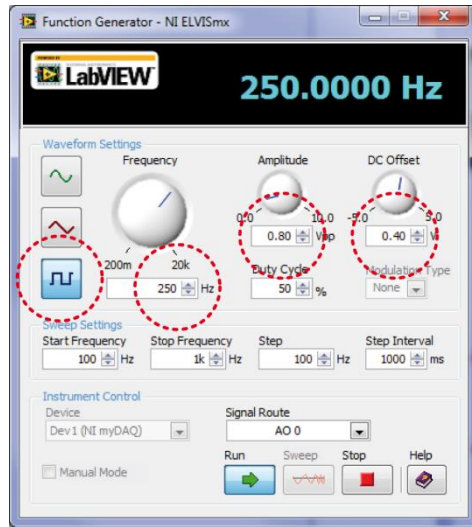
- Select the square wave.

- Set the **Frequency** to 250 Hz.

- Set the **Amplitude** to 0.8 V.

- Set the **DC Offset** to 0.4 V.

- Click **Run**.

This means that the signal generator is applying a voltage of 0.8 V for the first half of each cycle (charging the capacitor) and a null voltage for the second half (discharging the capacitor).

Step 6. Set the oscilloscope as follow:
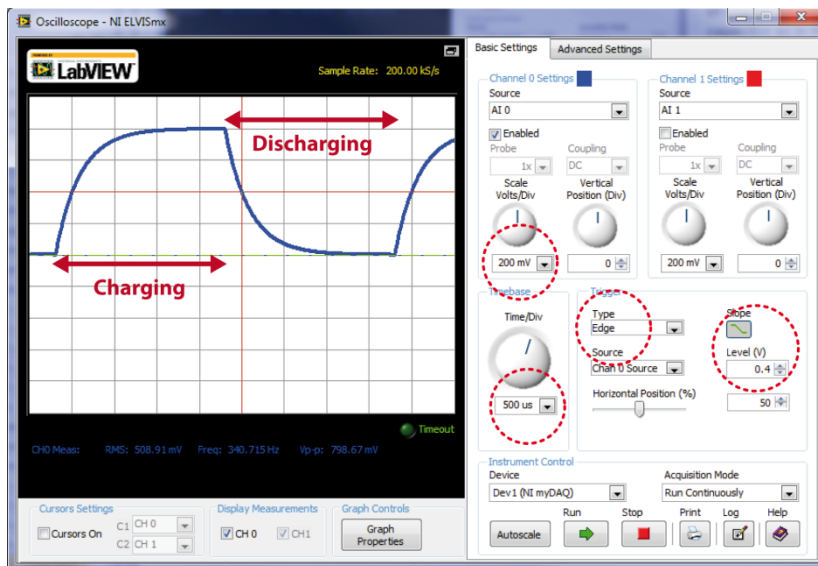
- Set the **Scale Volts/Div** to 200 mV.

- Set the **Time/Div** to 500 µs.

- Set the **Trigger Type** to Edge.

- Select the negative slope and set the **Level (V)** to 0.4.

- Click **Run**.

You should observe a complete cycle of charge and discharge of the capacitor as in the example on the left.

Step 7. Click **Stop** then click on the **Log** button in the bottom right corner of the oscilloscope to export the Voltage vs. Time data displayed on the oscilloscope.

Step 8. **Download and open the Logger Pro template from Brightspace**. Click **File/Import From/CSV…**. Select the file you just exported from the oscilloscope (make sure you click "all files" to find your text file). You should now see you charge/discharge voltage cycle.

Step 9. The y-axis represents a voltage but the x-axis must be formatted. The values for the time axis have been created for you in the template (Data Set - Time). To change the x-axis of your graph to the Time column, click on the x-axis on your graph and select more. In the pop-up window select Time (seconds) for the axis. Under scaling, select Autoscale then press OK.

Step 10. **Select the discharge region on your graph**. Click **Analyze/Curve Fit….** In the list of general equations, select **A*exp(-Ct)+B**. Click **Try Fit** then **OK**.

Step 11. Record the value of the **C** parameter from your fit. Note that "**C**" does not refer to the capacitance; it is just a fit parameter as **A** and **B** used in the fit formula. You do not have to print that graph but you should show it to your TA to make sure you did the right manipulation of the data.

Step 12. Use the **C** parameter and equation 5 to calculate the capacitance value of the capacitor in your circuit.

Cleaning up your station

Step 1. If you locally saved your files, send them to yourself by email. Pick up your USB key if you used one to save your files. Turn off the computer.

Step 2. Turn off the Fluke multimeter. Disassemble your circuit and put all wires, the three resistors, and the two capacitors back in the wire kit.

Step 3. Please recycle scrap paper and throw away any garbage. Please leave your station as clean as you can.

Step 4. Push back the monitor, keyboard and mouse. Also please push your chairs back under the table.