

Projectile motion

Introduction

In this experiment, a projectile launcher is used to investigate important concepts in two-dimensional kinematics. A steel ball placed in the launch barrel can be projected for different initial velocities, v_0 , and launch angles, θ . To predict where a projectile will land when it is shot from the launcher at some angle above the horizontal, it is first necessary to determine the initial speed of the projectile. The speed can be determined by shooting the ball and measuring a time using a pair of photogates. The two photogates that are positioned within the launch chamber allow for precise determination of the ball's launch speed. A software records when the ball passes through the first gate and the second gate. The program then calculates the difference, known as the pulse time. The average speed of the ball is then determined from the ratio of the separation distance of the photogates and the pulse time.

Projectile range

To predict the range, x , of the ball when it is shot with an initial speed, v_0 , at an angle, θ , above the horizontal, first predict the time of flight using the equation for the vertical motion:

$$y = y_0 + (v_0 \sin \theta)t - \frac{1}{2}gt^2, \quad (\text{eq. 1})$$

where y_0 is the initial height of the ball and y is the vertical position of the ball when it hits the target (see [Figure 1](#)). If we know v_0 and θ , we can solve this quadratic equation to find the flight time, t . We can then use $x = (v_0 \cos \theta)t$ to predict the horizontal range, x , of the projectile.

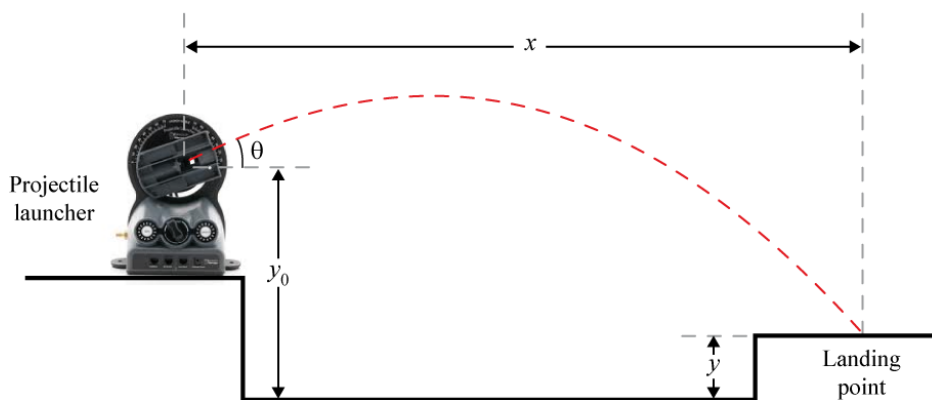


Figure 1 – Geometry of the projectile motion

Conservation of energy

The total mechanical energy of a projectile is the sum of its gravitational potential energy and its kinetic energy. In the absence of friction, total mechanical energy is conserved. When a projectile is shot straight up ($\theta = 90^\circ$), the initial gravitational potential energy (GPE) can be defined as zero. The initial kinetic energy, K , depends on the mass, m , of the projectile and the initial speed:

$$K = \frac{1}{2}mv_0^2, \quad (\text{eq. 2})$$

When the projectile reaches its maximum height, h , the speed of the projectile is zero and therefore the kinetic energy is zero. The maximum gravitational potential energy, U , depends on the mass of the projectile and the height:

$$U = mgh, \quad (\text{eq. 3})$$

where g is the acceleration due to gravity. If friction in the form of air resistance is ignored, the initial kinetic energy should equal the final gravitational potential energy.

Suggested reading

Students taking	Suggested reading	
PHY 1121	Section 3.3	Young, H. D., Freedman, R. A., <i>University Physics with Modern Physics, 14th edition</i> . Addison-Wesley (2014).
PHY 1321/1331 PHY 1124	Section 4.3	Serway, R. A., Jewett, J. W., <i>Physics for Scientists and Engineers with Modern Physics, 9th edition</i> . Brooks/Cole (2013).

Objectives

- ✓ Use photogates to determine the initial speed of the ball.
- ✓ Predict and verify the time of flight of a ball launched at an angle.
- ✓ Confirm that the initial kinetic energy of a projectile shot straight up is transformed into an equal amount of gravitational potential energy.
- ✓ Predict the landing point of a projectile in order to hit a target.

Materials

- Computer equipped with *Logger Pro* and a Vernier computer interface
- Launcher, hand pump and one steel ball
- Safety goggles
- Time of flight pad mounted on a lab jack
- Level
- Meter stick and universal support
- Electronic balance (one per classroom)
- 2 m stick or measuring tape
- Wooden box and foam mat

Safety warnings

Never look down the front of the barrel because it may be loaded. Safety glasses are provided, we recommend wearing the goggles at all times during data collection.

References for this manual

- *Projectile launcher*. PASCO scientific.

Procedure

Levelling the launcher

- Step 1.* Familiarize yourself with the projectile launcher. Make sure the launcher is installed on a sturdy table and secured with table clamps.
- Step 2.* Using the lower knob on the back of the unit, adjust the orientation of the launch chamber until level and secure the knob. Your launcher should aim towards your end of the table and not at the neighbouring student station.
- Step 3.* Using the upper knob on the back of the unit, adjust the position of the angle markings until the zero marking is aligned with the centre of the launch chamber and secure the knob.
- Step 4.* Now that you've aligned the zero marking with true horizontal, you can loosen the lower knob and move the launch chamber to the desired launch angle, and secure the knob.

Basic launching procedure

- Step 1.* Turn on your computer and launch the Logger Pro program.
- Step 2.* Connect the hand pump to the launcher (on its left side).
- Step 3.* Set the desired launch angle to $\theta = 45^\circ$.
- Step 4.* Insert the steel ball into the barrel. You might have to slightly push the ball at the entrance of the barrel before you it slowly going down to the bottom of the barrel by itself.

REMINDER: all students should wear goggles during the shootings.

- Step 5.* A wooden box has been installed at the end of the table to stop the ball. One student should stand near the end of the table to catch the ball after it hits the surface of the table and the box. Install the foam mat in front of the wooden box where the ball should land.
- Step 6.* Pump the hand pump until the pressure stabilizes. You should hear a small release sound when that pressure is reached. We recommend listening for at least three small release sounds and then waiting for five seconds to ensure the pressure has fully stabilized. The pressure should be around 40-50 psi. If the pressure is too low or too high, set the release pressure by adjusting the **Range knob**. Turn clockwise for higher pressure and counter-clockwise for lower pressure.
- Step 7.* Start data collection. Press and hold the **Arm button**, while still pressing the **Arm button**, press the **Launch button** to launch the steel ball. Data collection will stop automatically after the launch.
- Step 8.* Observe the range of the ball. We want the ball to land on the table and around 30 cm away from the end of the table. If the range is too short or too long, set the release pressure by adjusting the **Range knob**. Turn clockwise for higher pressure (higher initial velocity) and counter-clockwise for lower pressure (lower initial velocity). Keep launching the ball and adjusting the range knob again until you get the right pressure (right range) and then never touch the range knob until the end of the experiment.
- Step 9.* The program should give a launch speed value. The data collected are coming from the two photogates positioned within the launch chamber. Record the data displayed by the Logger Pro program in [Table 1](#). Use the data you collected to explain how the program calculates the initial speed. Note that the

distance between the two photogates is 5 cm.

Measure the launch velocity

- Step 1.* Launch the ball and record the launch speed in [Table 2](#).
- Step 2.* Collect four more readings to complete [Table 2](#).
- Step 3.* Determine the average speed and its standard error (bottom of [Table 2](#)). You just characterized the initial velocity at the pressure set by the range knob. It is important to not change the range knob until the end of the experiment or you will have to repeat this section to know the new initial velocity of the ball.

Predict the time of flight from the launch velocity

Once the launch speed of the projectile is known, you can use your knowledge of two-dimensional kinematics to predict where the projectile will land and its time of flight. You already know the approximate location of the landing point near the end of the table but you will now determine it from the measured launch velocity.

- Step 1.* Locate the time of flight pad. The pad functions as an electrical switch to provide timing information. When a projectile strikes the surface of the pad, a circuit is closed and timing information can be recorded by an interface and software. You will use this pad to measure the time interval taken by a projectile from the moment of launch to the moment of landing—in other words, the time of flight. Place the pad on the lab jack and adjust the jack height in order to have the surface of the pad at 0.146m from the surface of the table. That height corresponds to the height of the launching point (the center of the circular part of the launcher, see [Figure 1](#)). This setup is a particular case of the projectile motion for which $y_0 = y$. This simplifies [eq. 1](#), leaving us with:

$$0 = (v_0 \sin \theta)t - \frac{1}{2}gt^2 , \quad (\text{eq. 4})$$

and a time of flight given by:

$$t = \frac{2v_0 \sin \theta}{g} . \quad (\text{eq. 5})$$

- Step 2.* Verify that the launch angle is still set to $\theta = 45^\circ$. Calculate the time of flight and the horizontal range. Do the proper error calculations. Consider only the uncertainties on v_0 (i.e., assume $\theta = 45^\circ \pm 0^\circ$).
- Step 3.* Remove the foam mat and position the time of flight pad at the calculated location.
- Step 4.* Click **Experiment** then **Data Collection....** Change the value so that data collection ends after 5 events instead of 4.
- Step 5.* Do a test launch with data collection. If your ball is not landing on the pad, try to figure out why. Discuss with your TA if you need help.
- Step 6.* Have a look at the time points collected by the software. Events should have been generated by the projectile passing through each of the two photogates and by the projectile striking the pad. That is, a launch concluding with a strike on the pad will generate a total of five gate transitions: a block/unblock pair from the each of the two gates, and a block event from the pad strike. Note that the speed displayed at the bottom of the screen is not the launch speed anymore. The launch speed can still be

found at line “3” of the column “Speed”. The flight time is the time at line “5” of the column “Time”.

Step 7. Record five flight times and complete [Table 3](#).

Step 8. Determine the average experimental flight time and its standard error (bottom of [Table 3](#)). Compare with your predicted flight time calculated from the initial velocity.

Attempt to hit a target

Step 1. You will now be challenged to launch a ball at a particular target set by your TA. Ask your TA to come to your station. Your TA will pick a value between 10 cm and 25 cm for your target height (the lab jack with the time of flight pad) and a value between $\theta = 50^\circ$ and $\theta = 70^\circ$ for your launching angle. Your TA should write the values in your report.

**DO NOT TAKE ANY SHOTS TO PRACTICE!
IF YOU ARE CAUGHT PRACTICING, YOU WILL GET ZERO FOR THIS SECTION.**

Step 2. Calculate the horizontal distance, x , where the target should be placed to be hit by the projectile. For this calculation, y_0 and y are not equal anymore and you need to solve a quadratic equation to find the time of flight. If the vertical distances are measured with respect to the table, the y value is the target height picked by your TA and the y_0 value is the height of the launch point of 0.146 m (indicated on the back of the launcher).

HINT: The solutions of the quadratic equation $ax^2 + bx + c = 0$ are $= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Step 3. Position the target and ask your TA to come back to your station. Your TA will add a target printed on paper on your time of flight pad.

Step 4. Fire your projectile three times. There are different zones on the target with different point values. You'll get the sum of the points for your three shots (but you won't get more than 2 points).

Conservation of energy

Step 1. Adjust the angle of the launcher to $\theta = 90^\circ$. Use the level to set it straight up.

Step 2. Practice to shoot the ball and catch it on its way down before it falls back on the launcher. Using the meter stick and the universal support, estimate the maximum height attained by the ball compared to the midpoint between the two photogates (i.e., approximately 2.5 cm above the launching point).

Step 3. Once you are good at this, do one straight up shot with data collection. Record the launch velocity of that shot and the maximum height of the ball. Estimate an uncertainty for the height.

Step 4. Measure the mass of the ball. Calculate the initial kinetic energy, K , of ball. Calculate the potential energy, U , of the ball at the maximum height (use your experimental value of h). Compare both values.

Cleaning up your station

- Step 1.* Put back together the steel ball, the level, the foam mat and the goggles near by the wooden box.
- Step 2.* Push back the time of flight pad towards the center of the table. Also push back the meter stick/support assembly.
- Step 3.* Do not try to move or unclamp the launcher or the wooden box.
- Step 4.* Recycle scrap paper and throw away any garbage. Leave your station as clean as you can.
- Step 5.* Push back the monitor, keyboard and mouse. Also please push your chairs back under the table.