# Chapter 22

- Gauss's Law

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# Gauss's law background

- We've seen how to calculate the electric field from a charge distribution using the integration method.

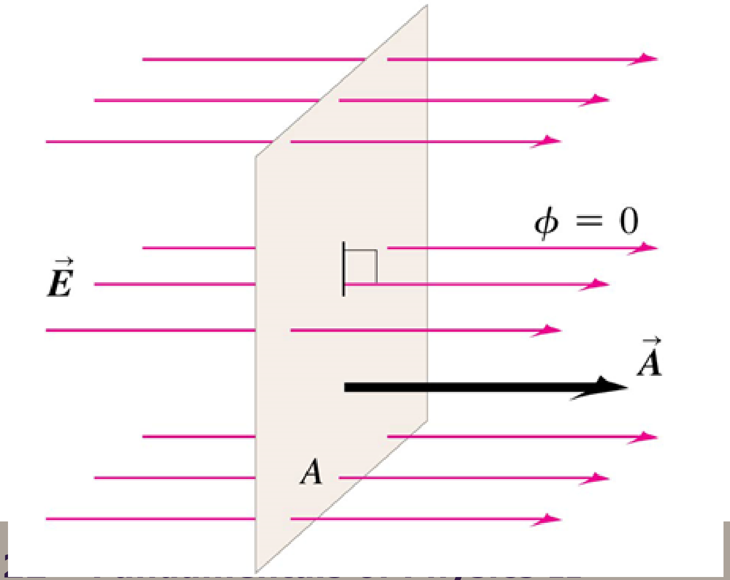
- Gauss's law is an alternative method.
  - Based on inverse-square behavior of electric force between point charges.
  - Can be useful for calculating electric fields of symmetric charge distributions (like infinite line, infinite plane, etc…)

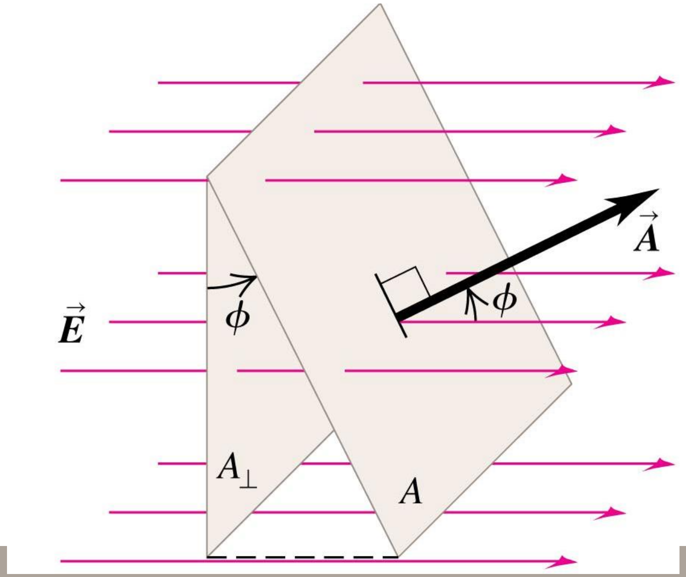
- Simple equation that involves electric flux and enclosed electric charge within a Gaussian "surface".

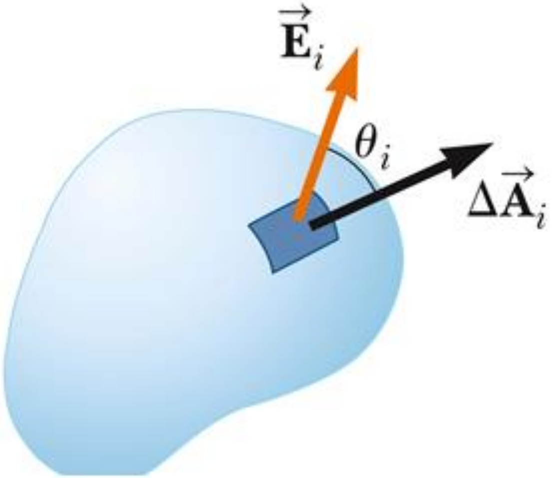
# Electric flux

- The **electric flux** is a measure of an electric field moving through a given surface area.
  - For flat surface, $\Phi_E = \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{A}}$
  - For a closed surface and nonuniform electric field:

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}}$$



# Gauss's law equation

- If given an electric charge (points or distributions), we can draw a **gaussian surface** that encloses all or part of the charge(s).

- The electric flux through the gaussian surface is equal to:

$$\boxed{\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0} = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}}}$$

  where $Q_{\text{encl}}$ is the charge that is inside the surface and $\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/Nm^2}$

- This allows us to calculate the electric field for **highly symmetric charge distributions** (when the integral is simple to calculate).

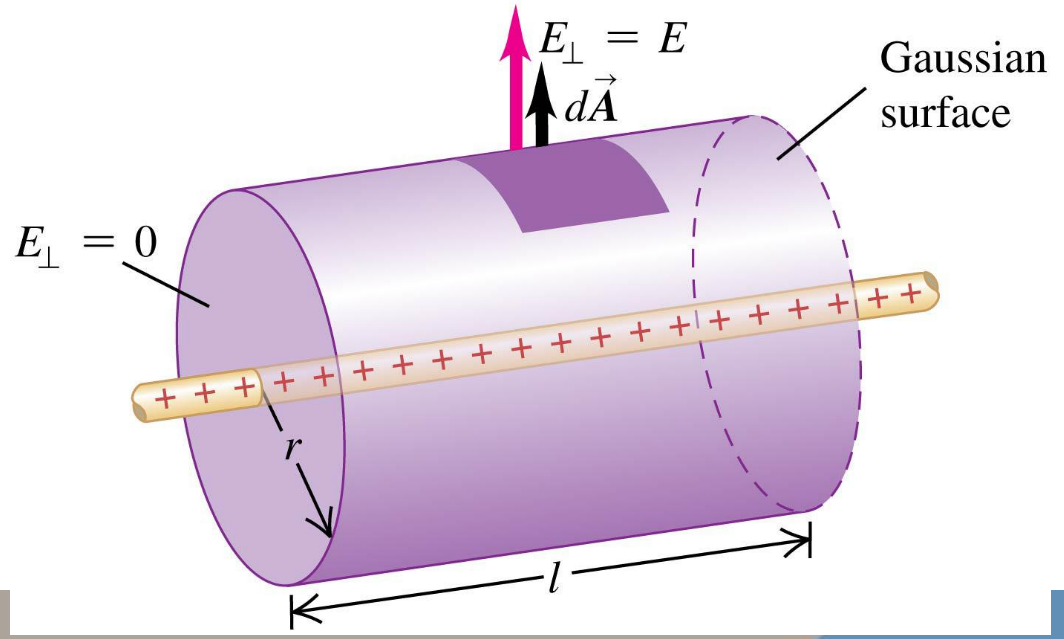
# Example: field of a uniform line charge

- Positive electric charge is distributed uniformly along an infinite long, thin wire with line charge density $\lambda$.

- We draw a gaussian surface in the shape of a cylinder that encloses a line of charge $l$. The integral $\int \vec{E} \cdot d\vec{A}$ is simple to calculate for the curved and flat parts of the cylinder.

- Using Gauss's law we obtain:

$$\boxed{E = \frac{2k\lambda}{r} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}}$$



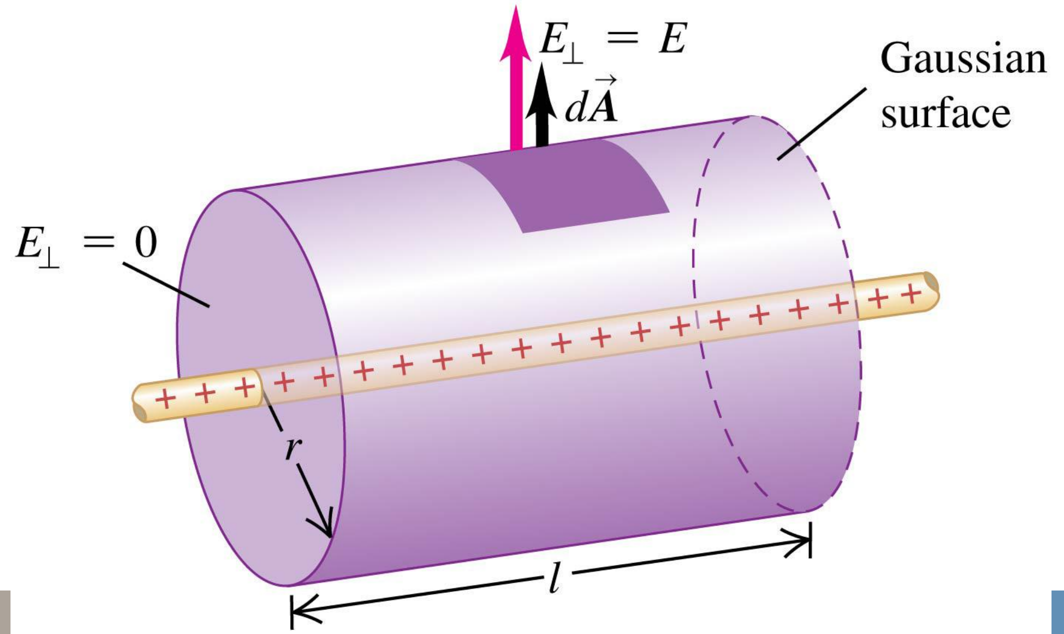
# Example: field of a uniform line charge

- Here's the math:

$$\Phi_{E,\text{curved}} = \int \vec{E} \cdot d\vec{A} = \int E\,dA\cos 0° = E\int dA = E(2\pi r)(l)$$

$$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0} = E(2\pi r l)$$

$$\boxed{E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}}$$



# Example: field of an infinite plane

- Positive electric charge is distributed uniformly across an infinite sheet with surface charge density $\sigma$.

- Here we use a cylinder that encloses a circle of charge with area $A$. Again the integral $\int \vec{E} \cdot d\vec{A}$ is simple to calculate.

- Our electric field using Gauss's law is:

$$\boxed{E = 2k\pi\sigma = \frac{\sigma}{2\epsilon_0}}$$

which represents a uniform field.