Chapter 24

- Capacitance and Dielectrics

Michael Wong – PHY 1122 Spring 2023

Learning goals

- What are capacitors and how do they store charge?
- Analysis of capacitors in a "network"
  - Series, parallel, combinations, etc…
- Calculation of energy in a capacitor.
- How dielectrics are used to make capacitors more effective.

Storing energy

- Pulling a rubber band or stretching an archer's bow: storing mechanical energy as *elastic potential energy*.
  - **Capacitors** store electric potential energy "in the electric field".

- Capacitors used in hundreds of day-to-day applications:
  - Flash photography, studfinder, defibrillator, etc…

- Capacitors are the first (of three) "simple circuit elements" we are introduced to.

Capacitors

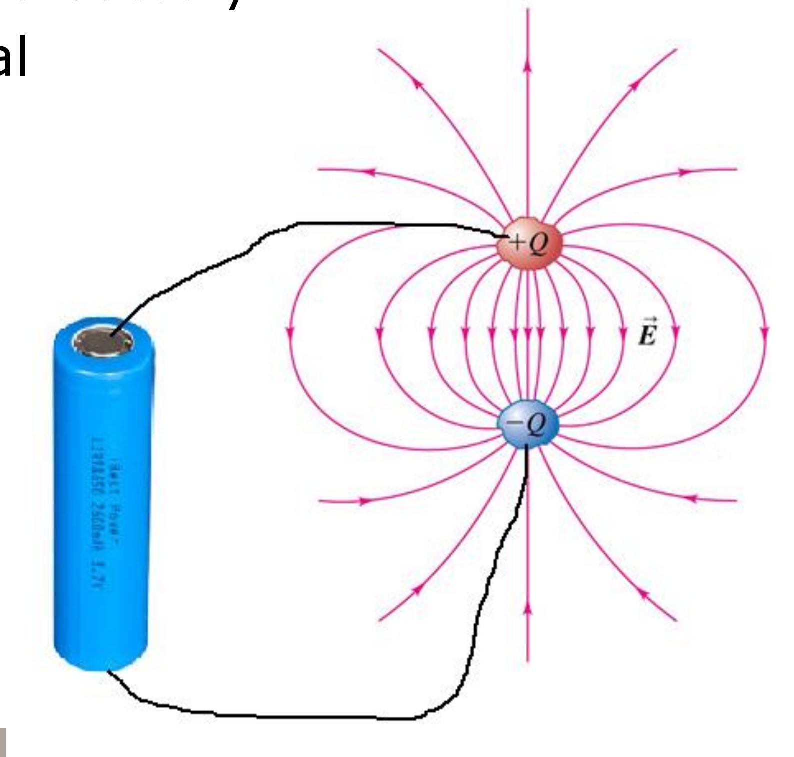
- To make and charge a capacitor: couldn't be simpler!
  Need 2 "separated" conductors and put equal and opposite charges on each.

- One method, attach each conductor to opposite ends of a battery.
  - The potential V_{ab} between the two conductors is equal to the battery's voltage.

- The **capacitance** C of the capacitor is:

$$\boxed{C = \frac{Q}{V_{ab}}} \qquad \text{unit: } 1\ \mathrm{F} = 1\ \frac{\mathrm{C}}{\mathrm{V}}$$

- Recall that V is U/q_0



Parallel-plate capacitor

- A **parallel-plate** capacitor has two parallel conducting surfaces with area A separated by distance d.

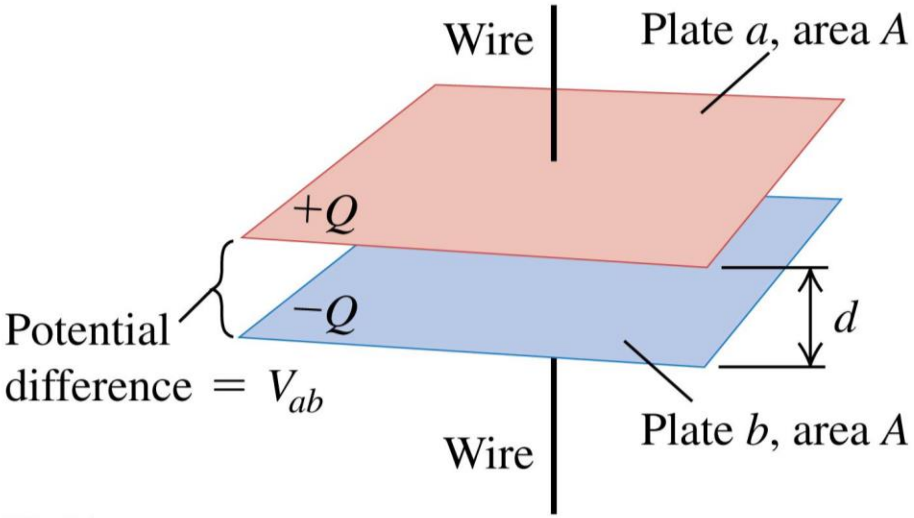
- The field is (more or less) uniform.

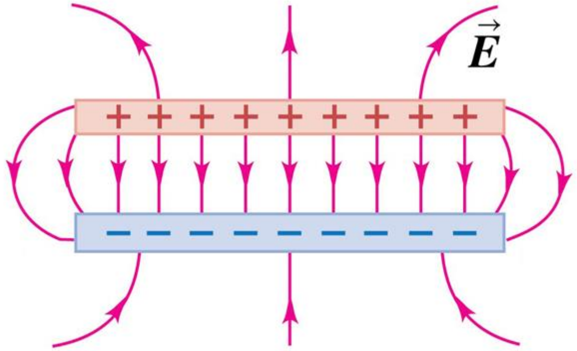
$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A}\frac{1}{\epsilon_0} \quad \rightarrow \quad V_{ab} = Ed = \frac{Qd}{\epsilon_0 A}$$

- The capacitance is:

$$\boxed{C_{\text{pp}}} = \frac{Q}{V_{ab}} = \boxed{\epsilon_0 \frac{A}{d}}$$

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\,\frac{\text{F}}{\text{m}}$$





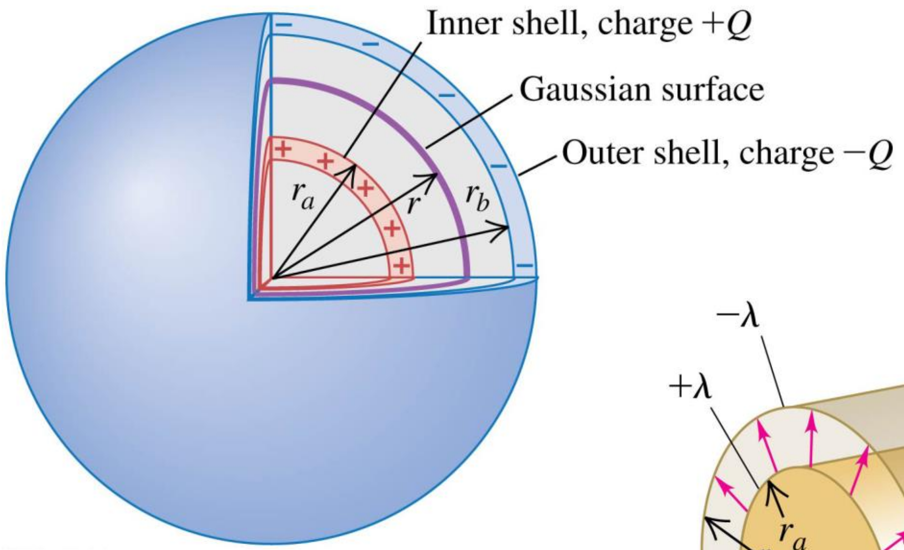
Different capacitances

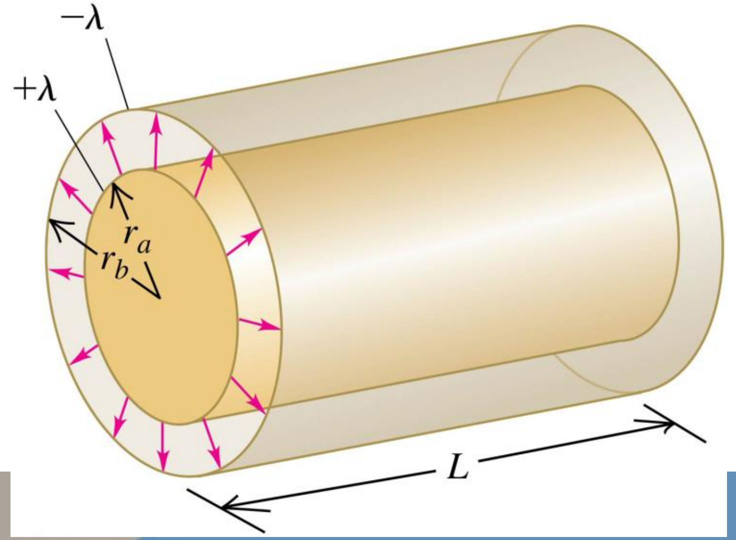
- The equations for capacitance for various geometries are shown in Ex. 24.2 – 24.4 (no need to solve).

- Parallel Plate: $$C = \epsilon_0 \frac{A}{d}$$

  Spherical: $$C = 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$$

  Cylindrical: $$C = \frac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}$$

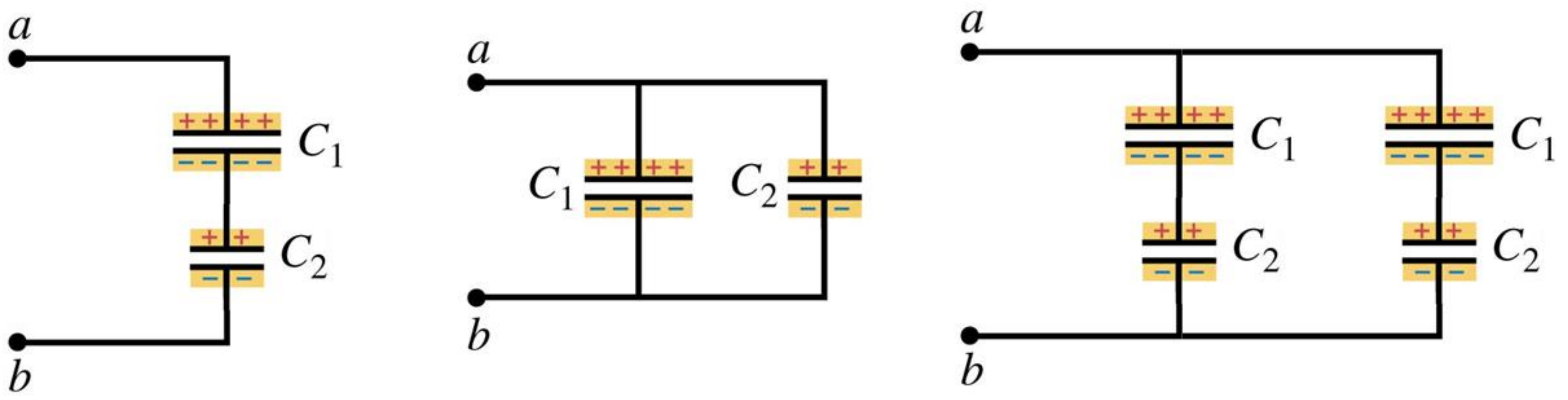




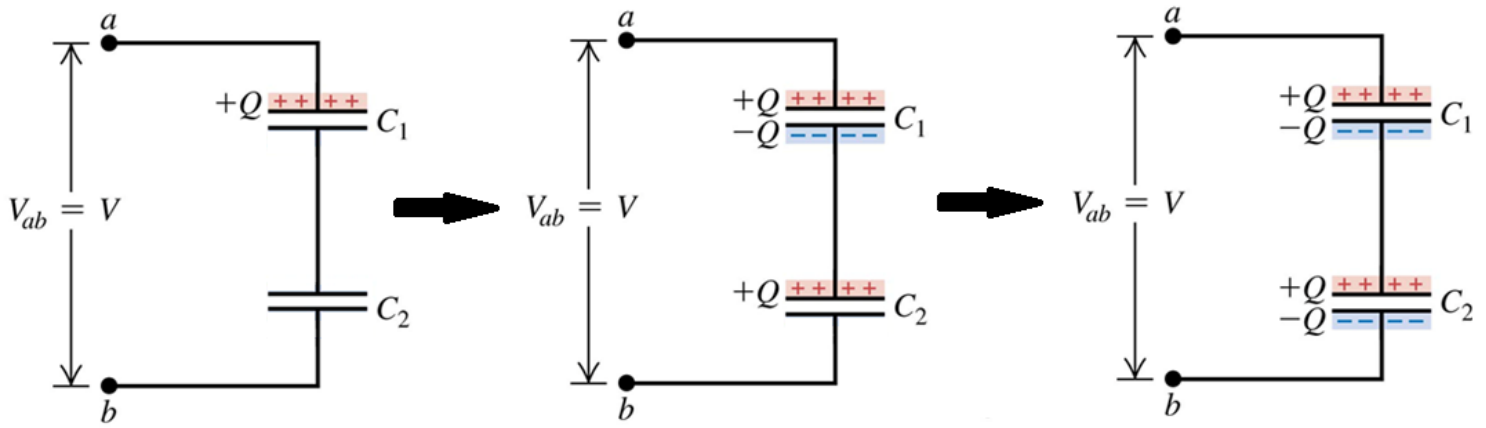
Networks of capacitors

- You can build networks of capacitors to suit your needs.
  - Depending on the connections (series, parallel, or combination) you can adjust the effective capacitance.

- Calculating effective capacitance based on series of rules.



Capacitors in series



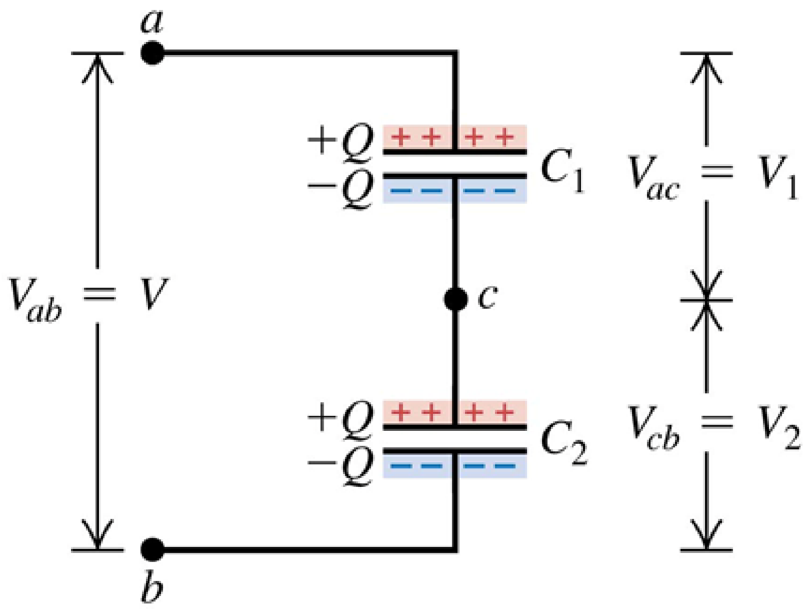
- Capacitors are in series if they are connected one after the other as shown.

- When a potential V_{ab} is applied, all plates will acquire the **same** charge of magnitude Q.

- The potentials from a to c and c to b are:

$$V_{ac} = V_1 = \frac{Q}{C_1}$$

$$V_{cb} = V_2 = \frac{Q}{C_2}$$



Capacitors in series

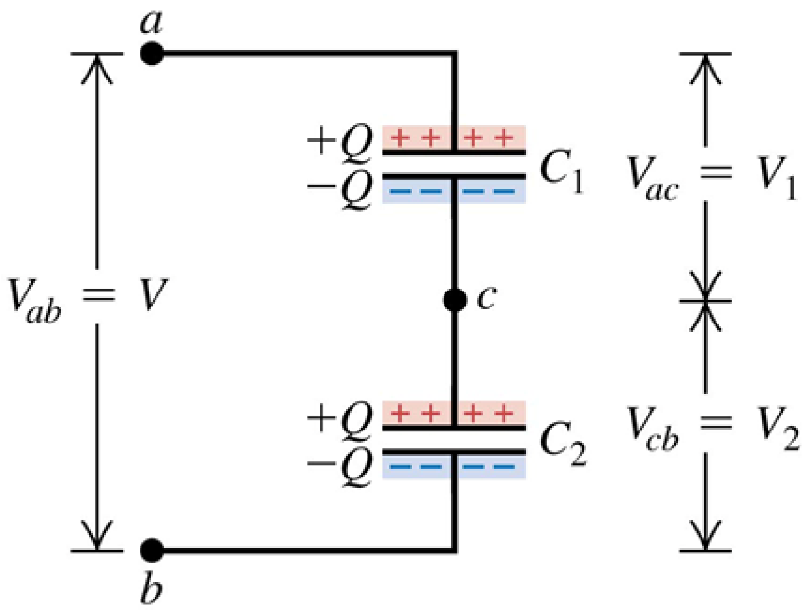
- The total voltage will be:

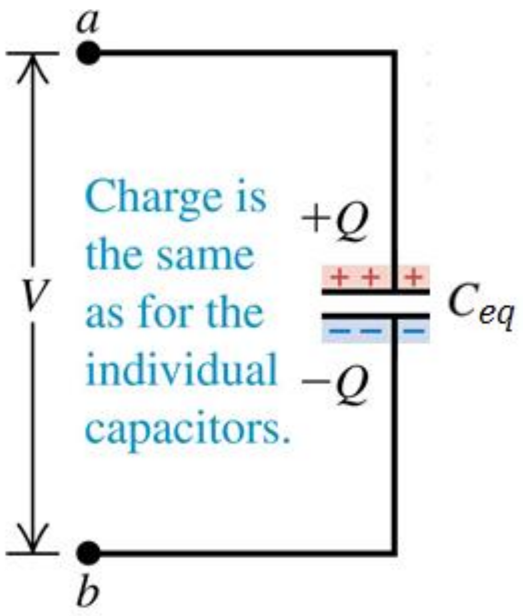
$$V = V_{ab} = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

- And we have the rule for adding capacitors in series:

$$\frac{V}{Q} = \boxed{\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_{eq}}}$$

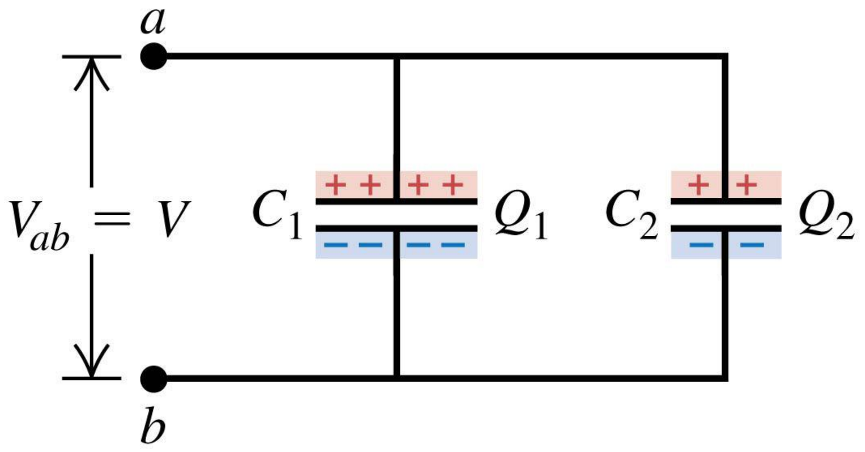
where C_{eq} is the equivalent capacitance of the combination. (Can be more than 2)





Capacitors in parallel

- Two capacitors are in parallel between a and b as shown below.

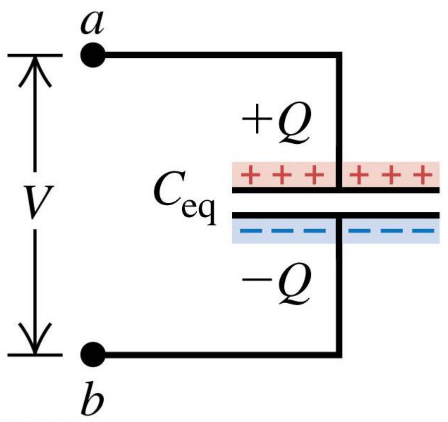


- The potential $V_{ab} = V$ is the same for both capacitors. The charges of each capacitor are thus: $\boxed{Q_1 = C_1 V}$ and $\boxed{Q_2 = C_2 V}$. The total charge of the equivalent capacitor is:

$$Q = Q_1 + Q_2 = V(C_1 + C_2)$$

- Therefore, we have:

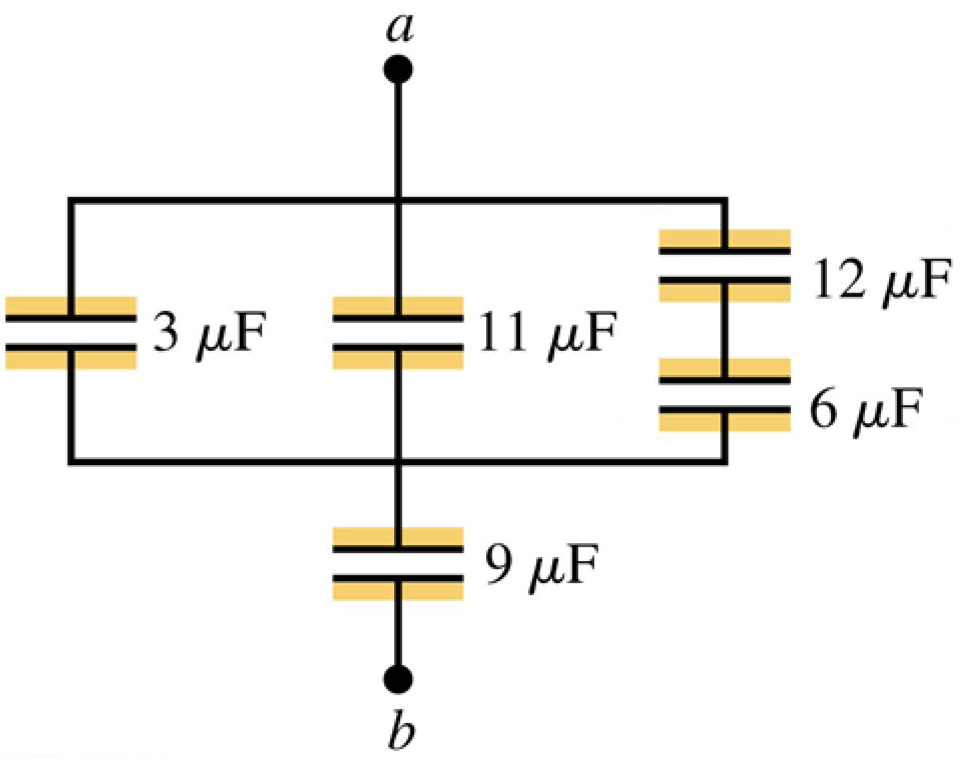
$$\frac{Q}{V} = \boxed{C_1 + C_2 = C_{eq}}$$



where C_{eq} is the equivalent capacitance for the parallel combination.

Ex. 24.6 – A capacitor network

- Find the equivalent capacitance of the five-capacitor network as shown in the figure.



$$3 + 11 + \left(\frac{1}{12} + \frac{1}{6}\right)^{-1} = 18$$

$$\boxed{C_{eq}} = \left(\frac{1}{18} + \frac{1}{9}\right)^{-1} = \boxed{6\ \mu\text{F}}$$

Energy stored in a capacitor

- Energy stored in a capacitor is equal to the work required to charge it. Capacitor discharge of energy is recovery of work.

- In a circuit with a battery, the energy is stored as chemical potential energy. Charging the capacitor converts it to electrical potential energy.

- The work required to charge is:

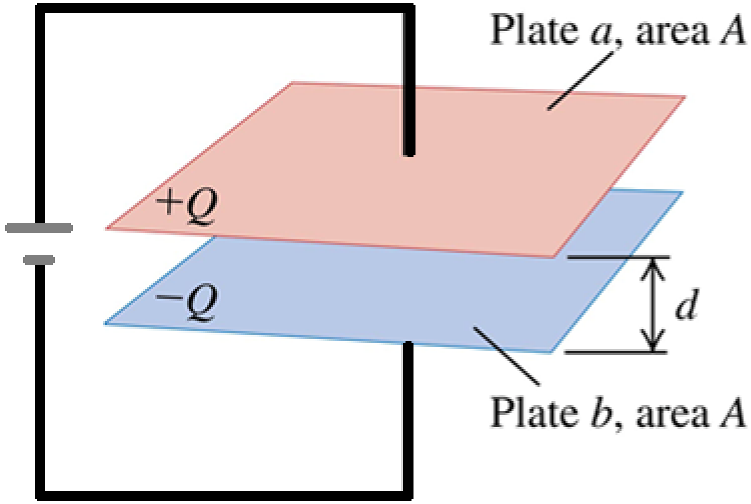
$$W = \boxed{U} = \int_0^Q \frac{q}{C} dq = \boxed{\frac{Q^2}{2C} = \frac{1}{2} QV = \frac{1}{2} CV^2}$$

  – The derivation comes from integrating the amount of work needed to move an element of charge dq through a potential difference of $v = \frac{q}{C}$.

Energy stored in a capacitor

- We consider the charging of the capacitor to be work done against the electric field. Therefore we define the energy density to be the stored energy per unit volume of the field.

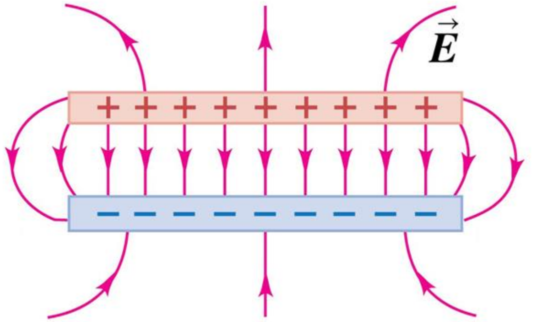
- For a parallel plate capacitor
$C = \frac{\epsilon_0 A}{d}$ and $V = Ed$.



- The energy density is:

$$\boxed{u} = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad} = \boxed{\frac{1}{2}\epsilon_0 E^2}$$

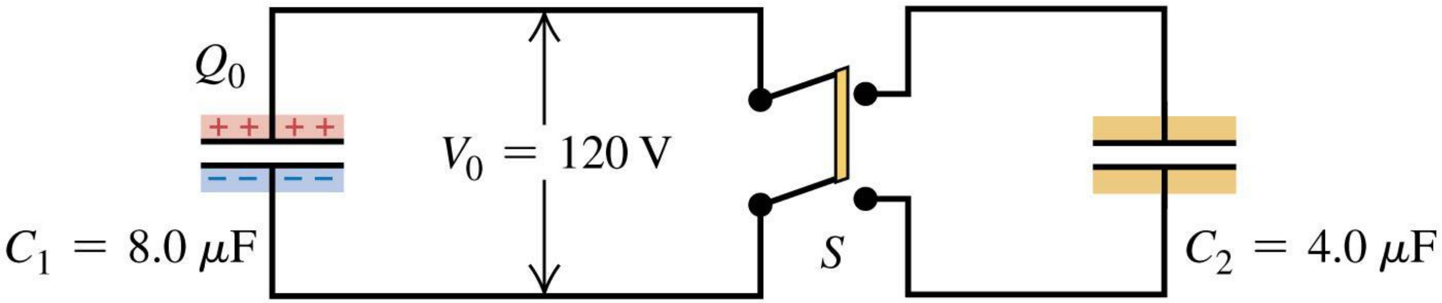
- This equation is valid for all capacitors.



Ex. 24.7 - Transferring Q and U

- We begin with capacitor $C_1 = 8.0\ \mu\text{F}$ charged to $V_0 = 120\ \text{V}$.
  (a) What is the charge Q_0 on C_1?
  (b) What is the energy stored in C_1?

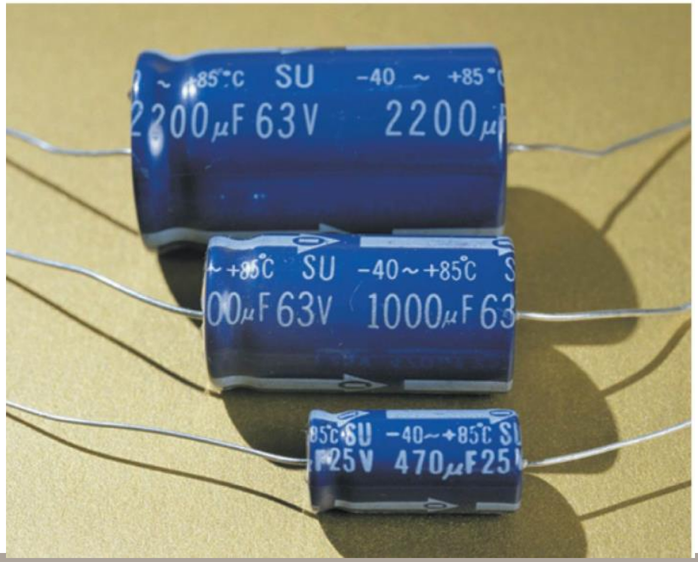
- We connect the system to capacitor C_2 by closing the switch shown in the figure. After charge no longer flows,
  (c) What is the potential difference across each capacitor and what is the charge on each capacitor?
  (d) What is the final energy of the system?

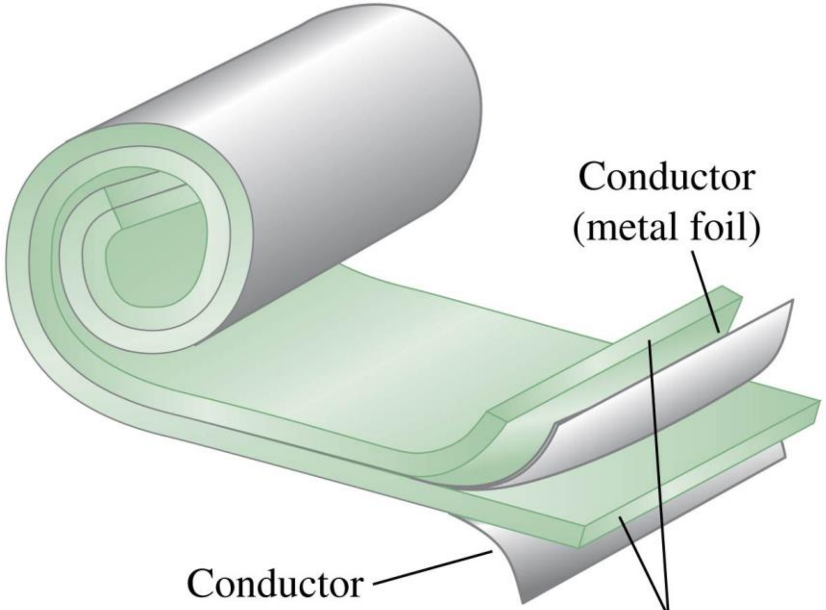


Dielectrics in capacitors

- The majority of capacitors have a insulating material (non-conducting, dielectric) between their plates.

- The figure shows strips of metal foil for plates separated by strips of plastic sheet (Mylar).





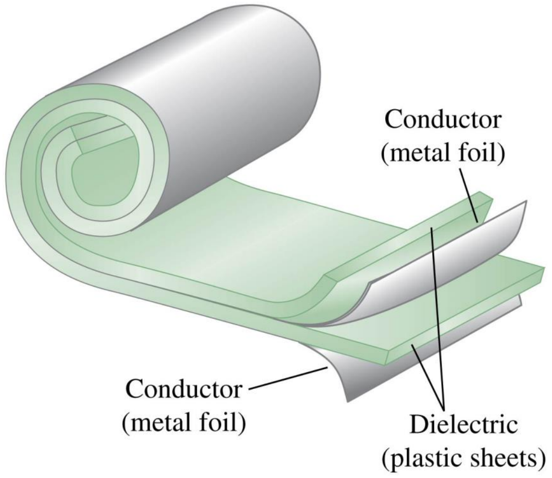
Dielectrics in capacitors

- Separating the conducting plates with a dielectric is useful for 3 reasons:

  1) Solves mechanical problem of keeping two large metal sheets separated.

  2) Increases the tolerance of **dielectric breakdown** between the conducting plates (maximum strength of field before charge jumps between the plates).

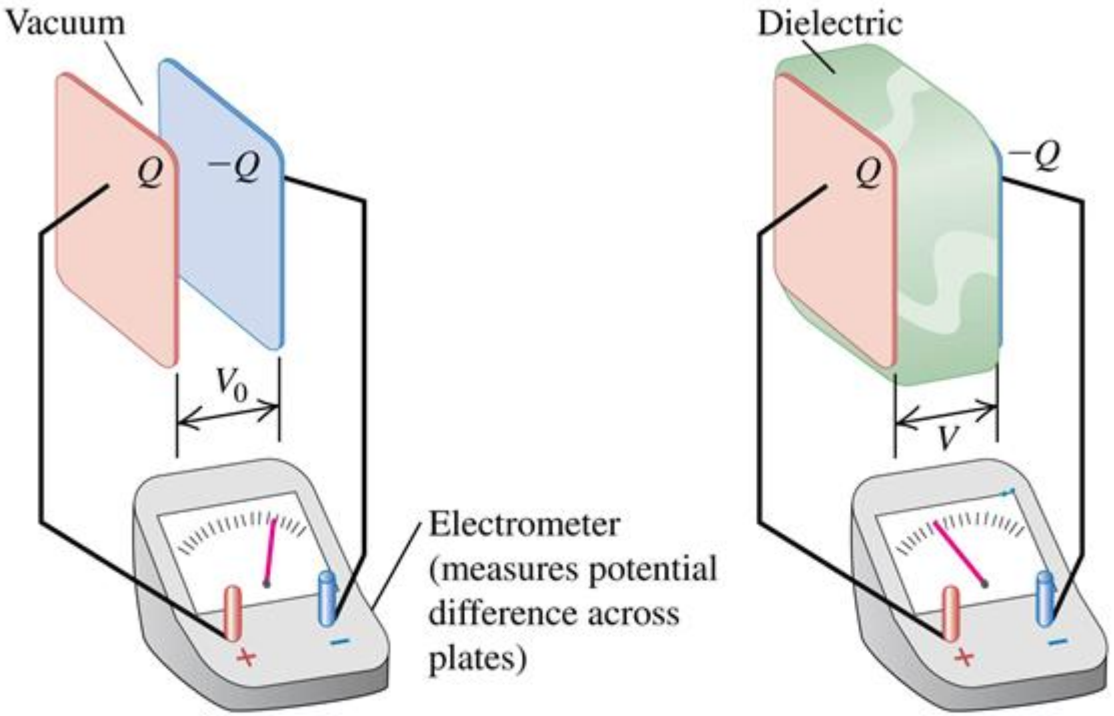
  3) Increases the capacitance!



Dielectrics in capacitors

- Consider an electrometer (voltmeter) between the two plates that measures potential difference V_0.

- With the dielectric, it measures V which is *less than* V_0.



Dielectrics in capacitors

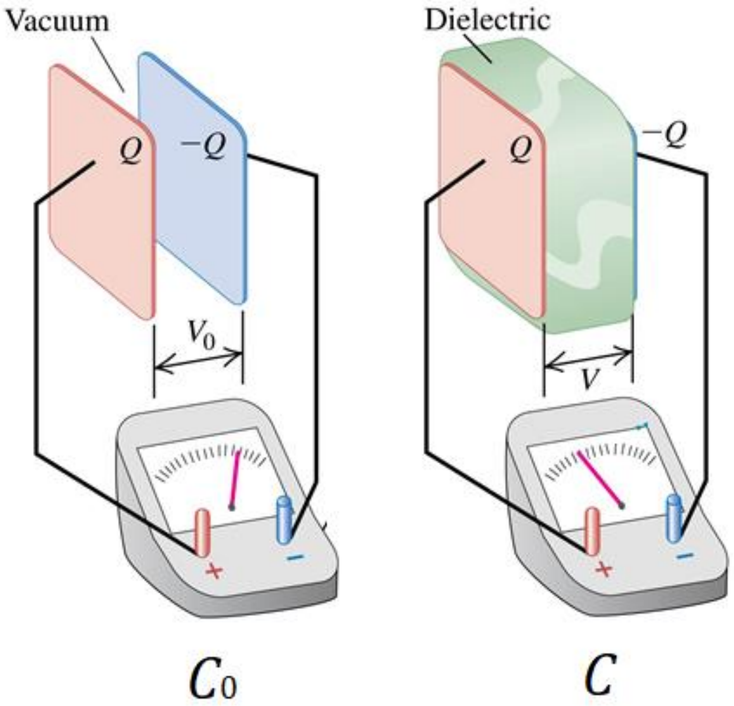
- Recall: $C = Q/V$.

- Adding the dielectric causes voltage to *decrease*. If charge remains the same then capacitance will *increase* from C_0 to C.

- The ratio of old to new capacitance is known as the **dielectric constant**, *K*.

$$\boxed{K = \frac{C}{C_0}}$$

- For vacuum, $K = 1.0$, air: $K = 1.0006$ plastics: $K = \sim 3 \text{ to } 6$, glass: $K = \sim 5 \text{ to } 10$



Induced charge and polarization

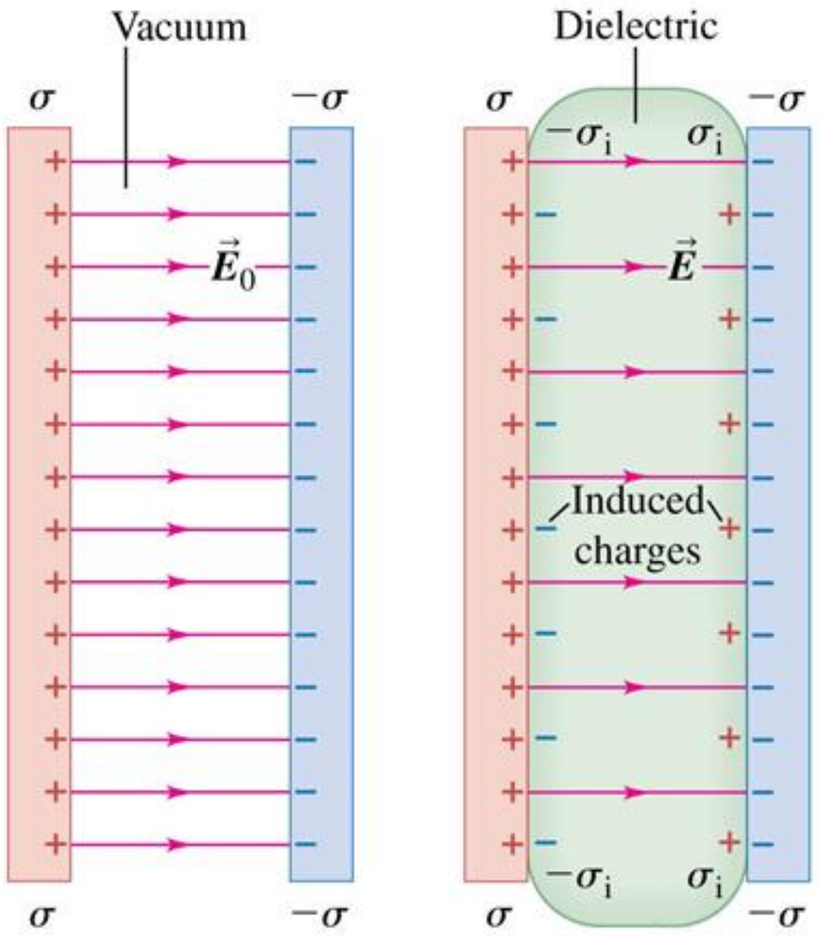
- The *decrease* in V when the dielectric is inserted is due to **polarization** of the charge in the dielectric (see figure).

- The induced surface charges σ_i (not necessary to calculate) reduces the electric field by:

$$E = \frac{E_0}{K}$$

- The **permittivity** of the dielectric is given by:

$$\epsilon = K\epsilon_0$$



Summary of dielectrics in capacitors

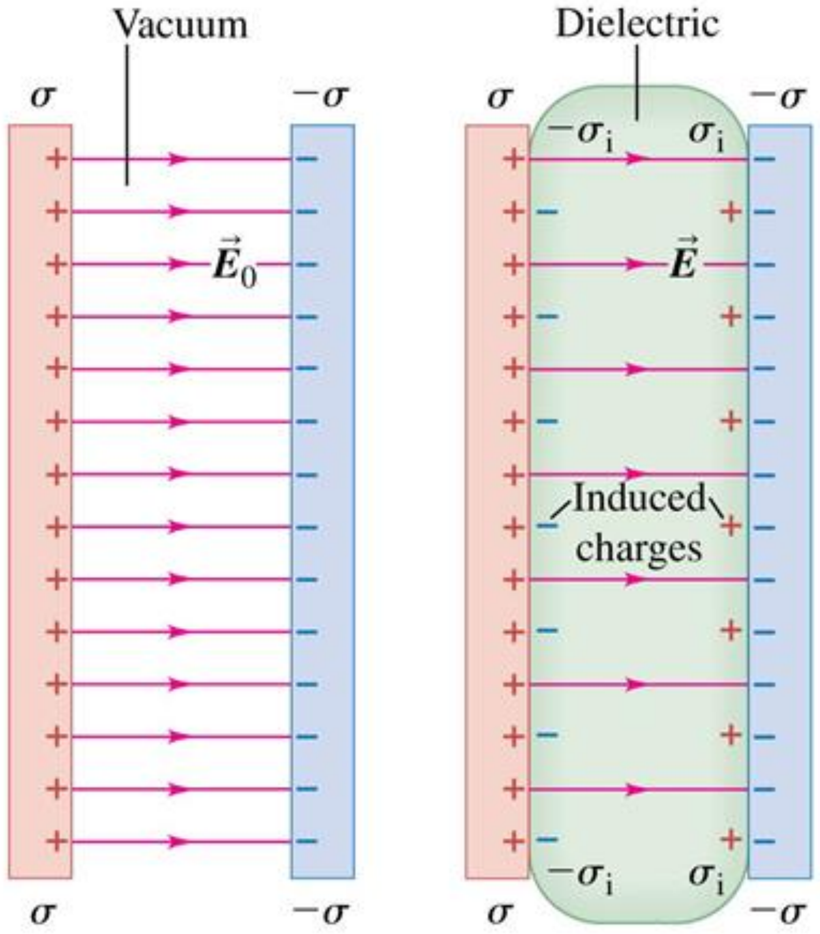
- The dielectric within the capacitor changes the following:

Increase in capacitance:	$C = KC_0$
Decrease in voltage:	$V = V_0/K$
Decrease in electric field:	$E = E_0/K$
Increase in permittivity:	$\epsilon = K\epsilon_0$
Decrease in electric energy:	$U = U_0/K$
Decrease in energy density:	$u = u_0/K$

Ex. 24.10/24.11 – Capacitor w and w/o dielectric

- Each parallel plates has area $A = 2000\ \mathrm{cm}^2$ and are a distance $d = 1.00\ \mathrm{cm}$ apart. It is charged to a potential difference $V_0 = 3.00\ \mathrm{kV}$ then disconnected. We insert an insulating plastic sheet between the plates and find the potential decreases to $V = 1.00\ \mathrm{kV}$ while the charge Q remains constant.

- Find C_0, Q, C, K, ϵ, E_0, and E (24.10).
- Find U_0, U, u_0 and u (24.11).



Dielectric breakdown and strength

- If the electric field is strong enough, **dielectric breakdown** occurs and the dielectric becomes a conductor.

- The **dielectric strength** is the maximum electric field the material can withstand before breakdown occurs.

Material	Dielectric Const., K	Dielectric Strength (10^6 V/m)
Air / Vacuum	1.006 / 1	3 / ---
Paper	3.7	16
Pyrex glass	5.6	14
Teflon	2.1	60
Styrofoam	2.56	24
Rubber	6.7	12
Sr. Titanate	233	8

Some common applications

- Camera flash
  - Energy stored is released when button is pushed creating bright flash of light

- Electronic stud finder
  - The device looks for metal fasteners behind a wall that act as dielectric and change capacitance values.

- Capacitive touchscreens on phones
- Computer keyboards
- Defibrillators