Chapter 23

- Electric Potential

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Learning goals

- Relationship between work and **electric potential energy**.
  - Calculations for point charge(s).
- Meaning of **electric potential**.
  - Calculations for point charge(s) and charge distributions.
- How to use **equipotential surfaces**.
- Calculating electric field from electric potential.

Electric potential energy

- Previously we connected electromagnetism to force, now we connect it to **energy**.
- In mechanics, conservation of energy used to solve problems you can't solve "*with force*". Same idea is applied here.
- Basis of our analysis: when a charge moves in an electric field, **work** is done.
  - Expressed in terms of electric potential (energy).

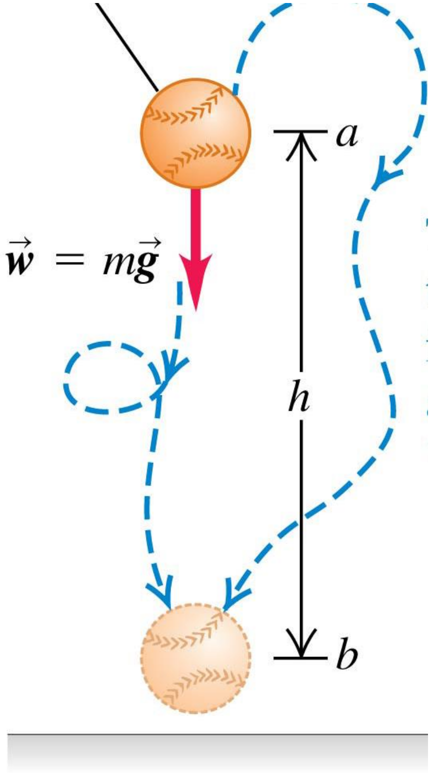
Recall: Work done by a force

- A force $\vec{F}$ acts on a particle and moves it from a and b.
- We express the work done in terms of potential energy:

$$W_{a\to b} = U_a - U_b$$

- Work can be negative (throwing ball straight up) or ***positive*** (ball falling back down).
- Can also look at it in terms of force x displacement:

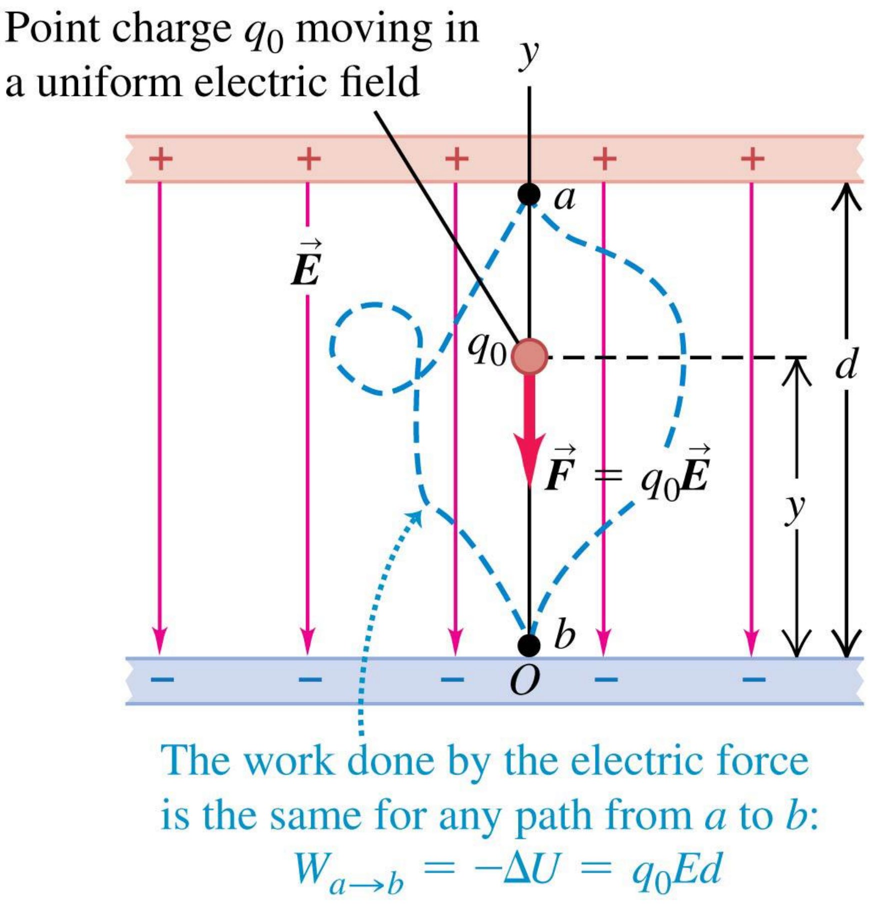
$$W = Fh = mgh$$



Electric potential energy in uniform field

- **Uniform** E-Field pointing down.
  - Exerts downward force on positive charge.
- As charge moves down, work done is independent of the path.
- Work is the electric force F multiplied by displacement d:

$$W_{a\to b} = Fd = q_0 E d$$



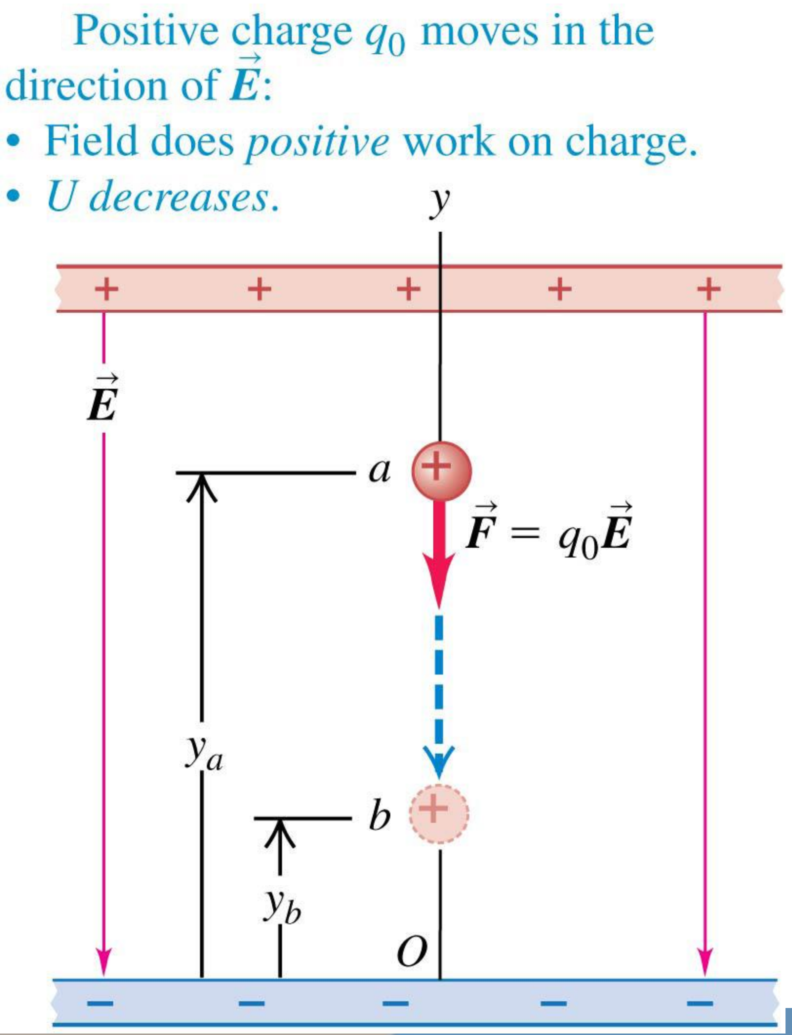
Electric potential energy in uniform field

- Positive charge moves in direction of field → field does *positive* work on the charge.
- Potential energy of the charge *decreases*.
- Equation for potential energy at height y:

$$U = q_0 E y$$

  (only for **uniform electric field**)
- Work to move from a to b:

$$W_{a\to b} = q_0 E (y_a - y_b)$$



Work in nonuniform field

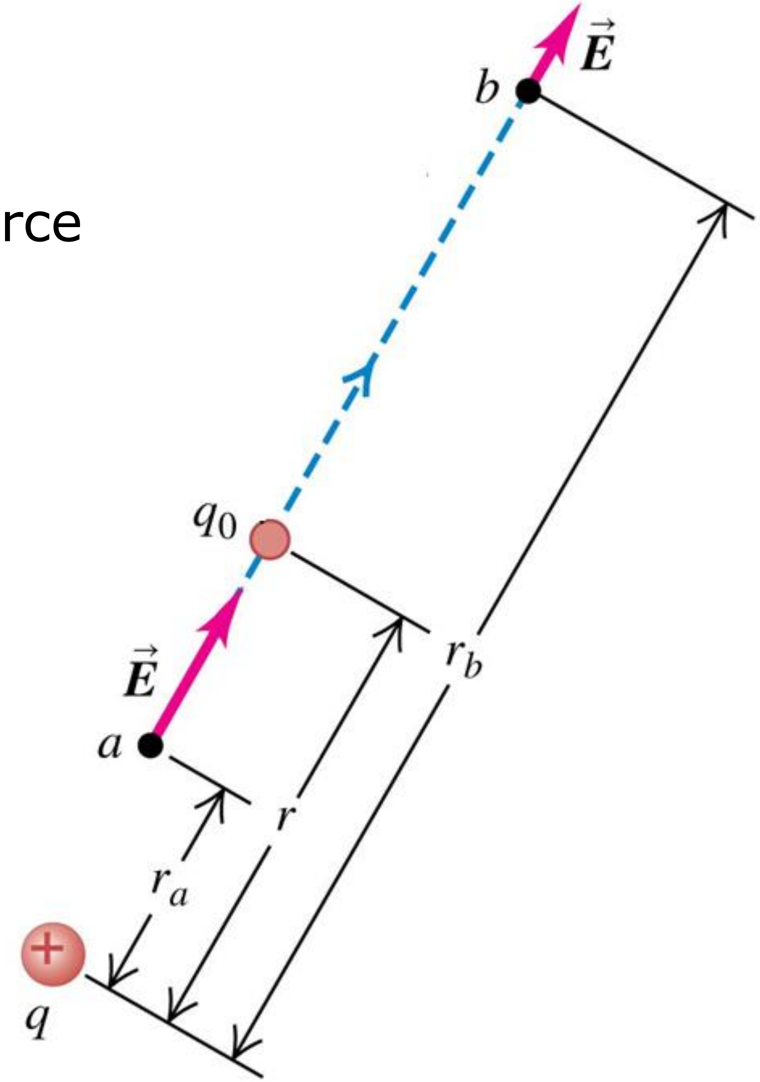
- For the case when the electric field is not uniform, our force varies with position.

- Consider the particle moving from a to b in the field of a point charge (along an electric field line).

$$W_{a\to b} = \int_{r_a}^{r_b} F_r dr = \int_{r_a}^{r_b} k\frac{qq_0}{r^2} dr = kqq_0\left(\frac{-1}{r_b} - \frac{-1}{r_a}\right)$$

$$\boxed{W_{a\to b} = kqq_0\left(\frac{1}{r_a} - \frac{1}{r_b}\right)}$$

- The work done again depends only on the pointa a and b compared to the charge q.

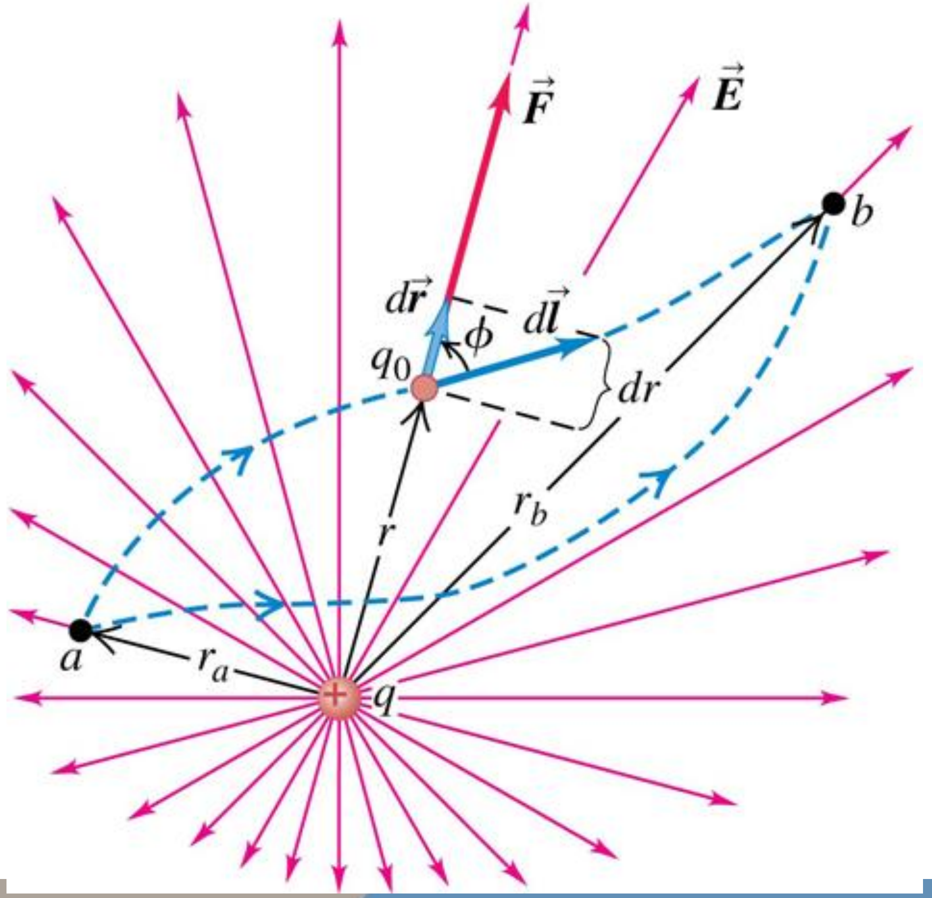


Work in non-uniform field and general displacement

- If the displacement is more general in a nonuniform electric field. Consider the charge moving from point a to point b.

- (1) The force $\vec{\boldsymbol{F}}$ is always in the direction of the field.
  (2) The differential displacement $d\vec{\boldsymbol{l}}$ is along the blue path.

$$W_{a\to b}=\int_a^b \vec{\boldsymbol{F}}\cdot d\vec{\boldsymbol{l}}=\int_a^b F\,dl\cos\phi$$



- The figure shows that $dl\cos\phi=dr$

$$W_{a\to b}=\int_a^b F\,dr=kqq_0\left(\frac{1}{r_a}-\frac{1}{r_b}\right)=U_a-U_b$$

$$\boxed{U_a=k\frac{qq_0}{r_a}}$$

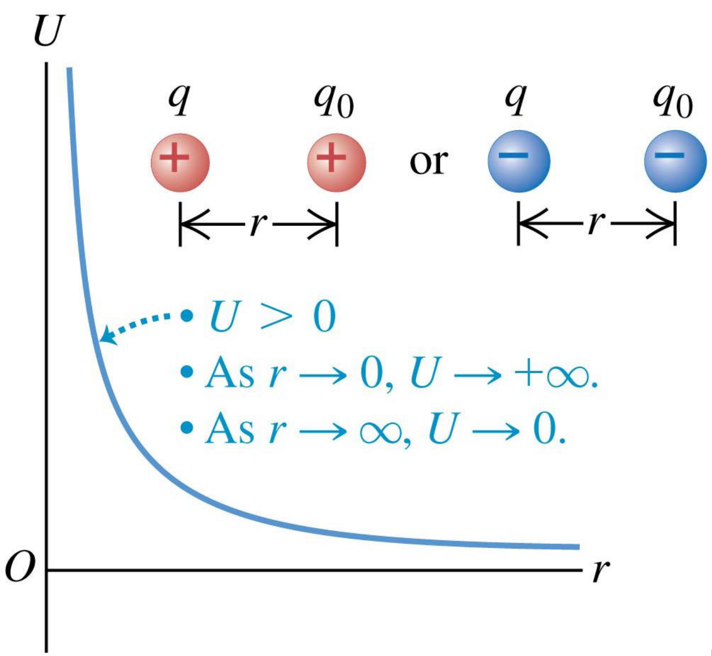
Depends **only** on position.

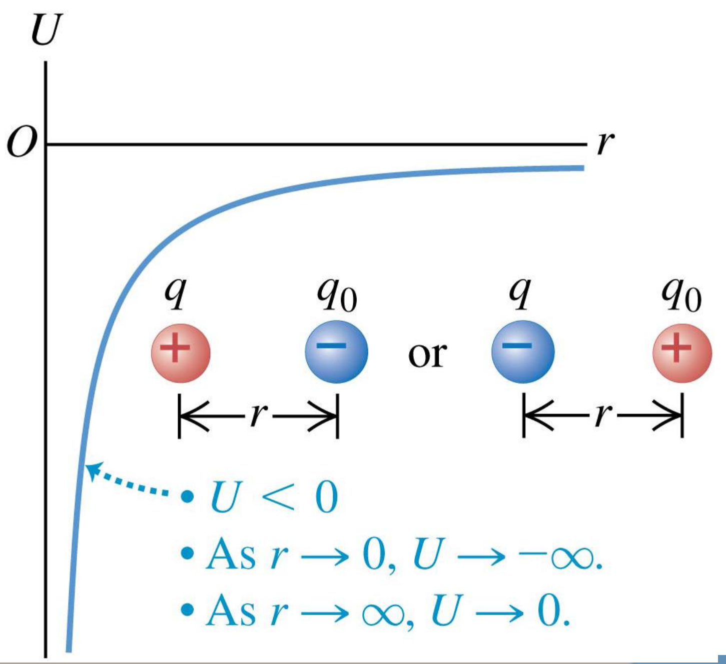
Potential energy vs. force

- The equations for potential energy and radial component of force that one charge exerts on the other are similar:

$$U = k\frac{qq_0}{r}, \qquad F_r = k\frac{qq_0}{r^2}$$

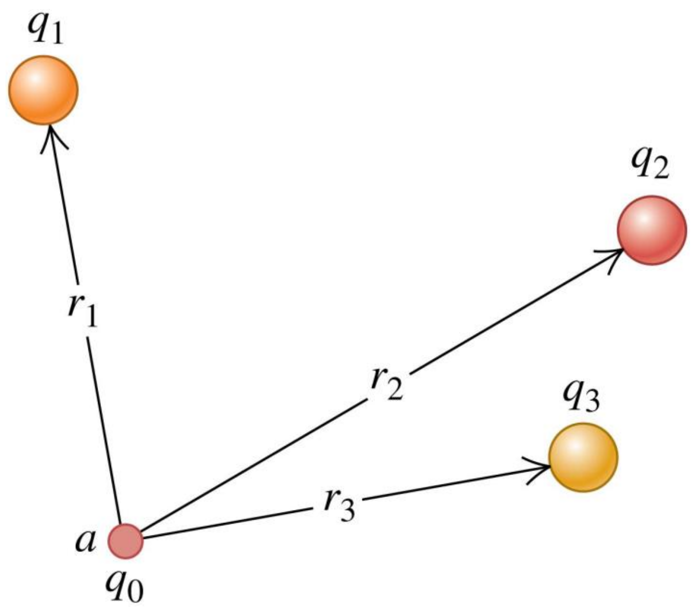
U depends on $1/r$ and F_r on $1/r^2$.





Electric potential energy (superposition)

- The potential energy associated with q_0 depends on the other charges in its vicinity and their distances from q_0.



- For multiple point charges q_1, q_2, q_3 the potential energy associated with q_0 is:

$$\boxed{U = kq_0\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)}$$

- Think of this as the energy it takes to bring q_0 from far away to the point a.

- The **total potential energy** to assemble all charge

$$\boxed{U_{\text{tot}} = k\sum_{i<j}\frac{q_i q_j}{r_{ij}}}$$

Ex. 23.2 – System of point charges

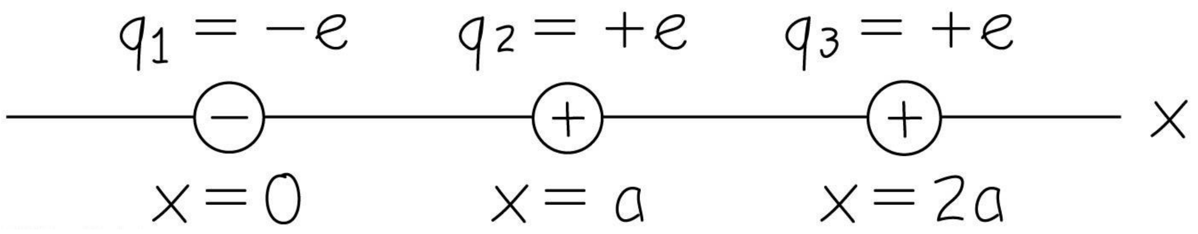
Two point charges are located on the x-axis:

$q_1 = -e$ at the origin

$q_2 = +e$ at $x = a$

(a) Find the work that must be done by an external force to bring a third point charge $q_3 = +e$ from infinity to $x = 2a$.

(b) Find the **total potential energy** of the three charges.



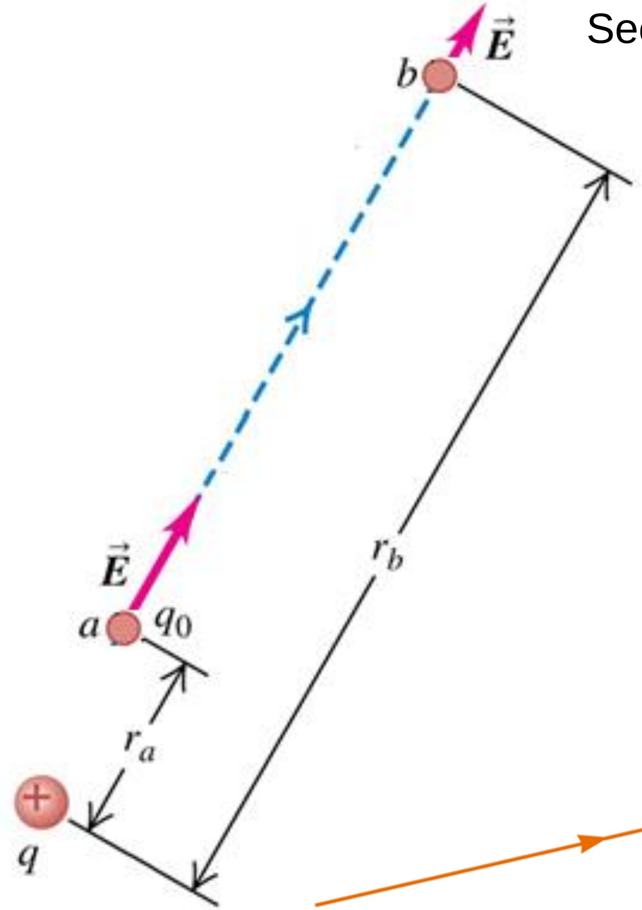
Electric potential

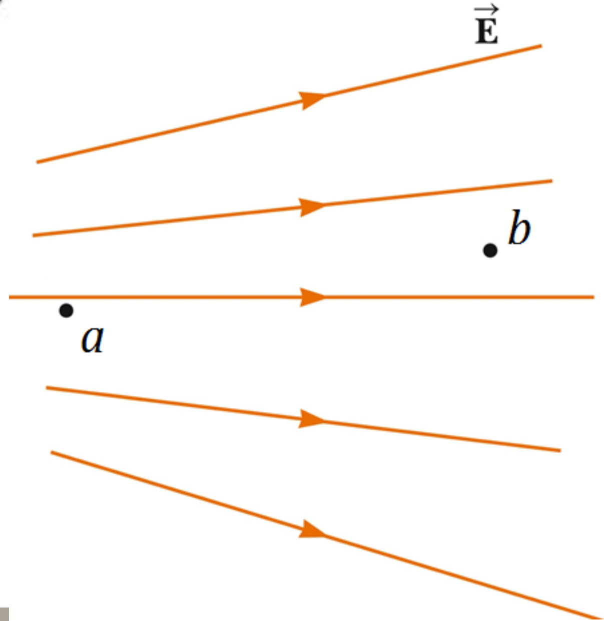
- The **electric potential** is the amount of *electric potenti* *energy per unit charge:* $\boxed{V = U/q_0}$
  - Recall that electric field is measure of electric force per unit charge: $\vec{E} = \vec{F}/q_0$
  - Electric potential is measured in volts (1 V = 1 J/C)

- Electric potential exists even due to a single charge (just like electric field).

- Consider the electric field to the right.
  - Put a test charge at a it will have energy U_a.
  - Electric potential at a is V_a. At b is V_b.

- It is the work needed to move a unit charge to a point in the field.

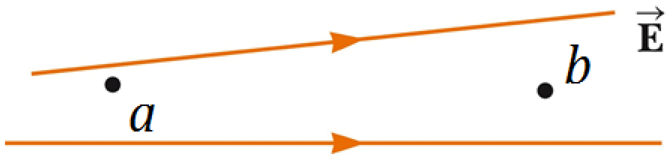




Electric potential difference

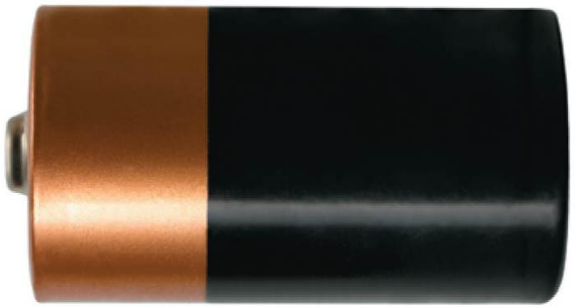
- The electric potential of a with respect to b ($V_{ab} = V_a - V_b$) is the work done by the electric force when a *unit* (1 C) of charge moves from a to b. This is the **potential difference**.

- As you move along the field, electric potential *decreases*: $V_a > V_b$



- Later in electric circuits, we will see potential difference between two points in a uniform electric field is called **voltage** (like for a power supply or battery).

Point a
positive
terminal



Point b
negative
terminal

$V_{ab} = 1.5$ volts

Calculating electric potential for a point charge

- Similar to field, potential can be measured due to a single body of charge:

$$\boxed{V} = \frac{U}{q_0} = \frac{k\frac{qq_0}{r}}{q_0} = \boxed{k\frac{q}{r}}$$

- For a collection of point charges or a charge distribution, electric potential is a sum or integration over the charges.

$$V = k\sum \frac{q_i}{r_i} \quad \text{or} \quad V = k\int \frac{dq}{r}$$

- Potential can be **positive or negative**, depending on charge(s).

Finding electric potential from the field

- In some cases, it's easiest to calculate V from $\vec{\boldsymbol{E}}$ (if field is known or easily found, like a charge distribution).

$$W_{a\to b} = \int_a^b \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = \int_a^b q_0 \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = U_a - U_b$$

$$\boxed{V_a - V_b} = \frac{U_a}{q_0} - \frac{U_b}{q_0} = \boxed{\int_a^b \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}}}$$

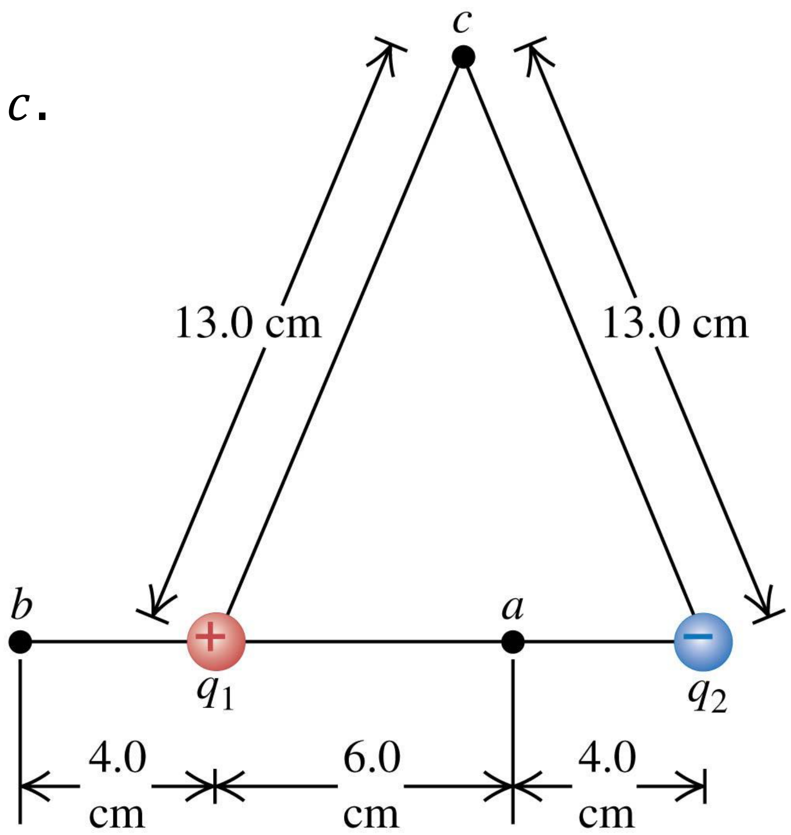
- Look at units: we can express the field in terms of volts:

$$1\ \frac{\mathrm{N}}{\mathrm{C}} = 1\ \frac{\mathrm{J}}{\mathrm{C}}\frac{1}{\mathrm{m}} = \boxed{1\ \frac{\mathrm{V}}{\mathrm{m}}}$$

so the electric field is a measure of the rate of change of the electric potential (V) with respect to position (m).

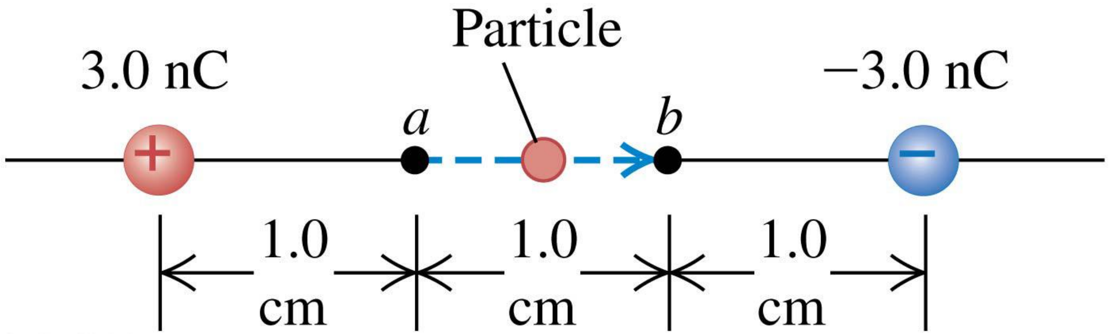
Exs. 23.4, 23.5 – Dipole

- An electric dipole consists of point charges $q_1 = +12\ \mathrm{nC}$ and $q_2 = -12\ \mathrm{nC}$ placed 10.0 cm apart.
- **Ex. 23.4**: Calculate electric potential at points a, b, and c.
  Hint: For potential, we simply add up the scalars.
- **Ex. 23.5**: Compute potential energy associated with charge $q = 4.0\ \mathrm{nC}$ placed at a, b, and c.
  Hint: Use your V from Ex. 23.4.



Ex. 23.7 – Moving through ΔV

- A dust particle with mass $m = 5.0 \times 10^{-9}\ \mathrm{kg}$ and charge $q_0 = 2.0\ \mathrm{nC}$ starts from rest and moves in a straight line from point a to point b between the two charges as shown in the figure. What is its speed v when it reaches point b?



- We can use the conservation of energy equation since the only force that acts on the particle is electric force.

Electric potential of charge distributions

- To calculate the electric potential due to a charge distribution, one of two methods should work depending on how the distribution looks.

  1) Use the equations to sum up the potentials or integrate over the total potential:

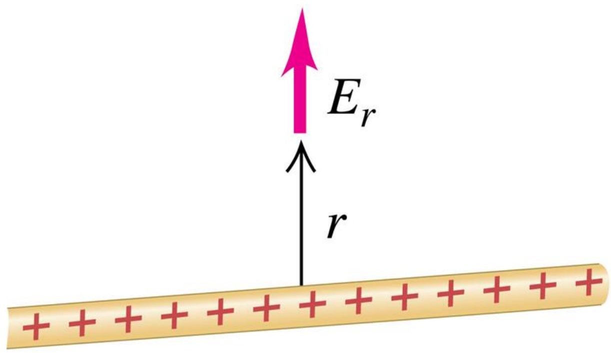
$$\boxed{V = k\sum_i \frac{q_i}{r_i}} \quad \text{or} \quad \boxed{V = k\int \frac{dq}{r}}$$

  2) If we know how electric field depends on position, use:

$$\boxed{V_a - V_b = \int_a^b \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}}}$$

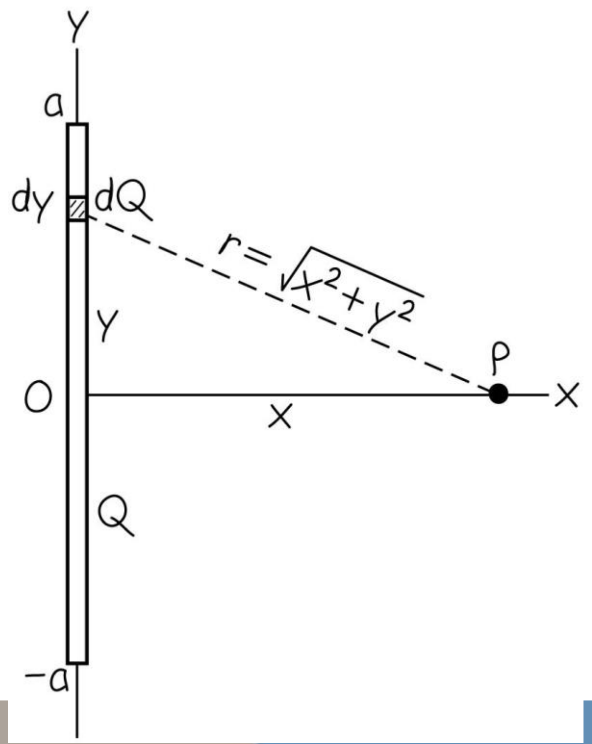
Ex. 23.10 – Infinite line of charge

- Find the potential at a distance r from a very long line of charge with linear charge density λ.



Ex. 23.12 – Finite line

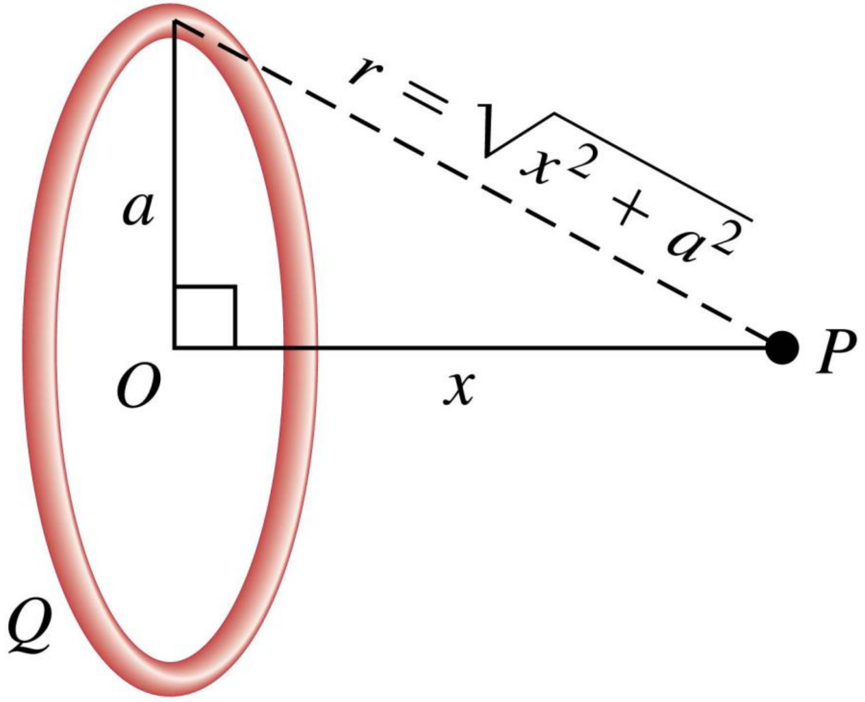
- Positive charge Q is distributed uniformly along a line of length $2a$ on the y-axis from $y = -a$ to $y = +a$. Find the electric potential at point P on the x-axis at distance x from the origin.



Ex. 23.11 – Ring of charge

- Electric charge Q is distributed uniformly around a thin ring of radius a. Find the potential at a point P on the ring axis a distance x from the center of the ring.

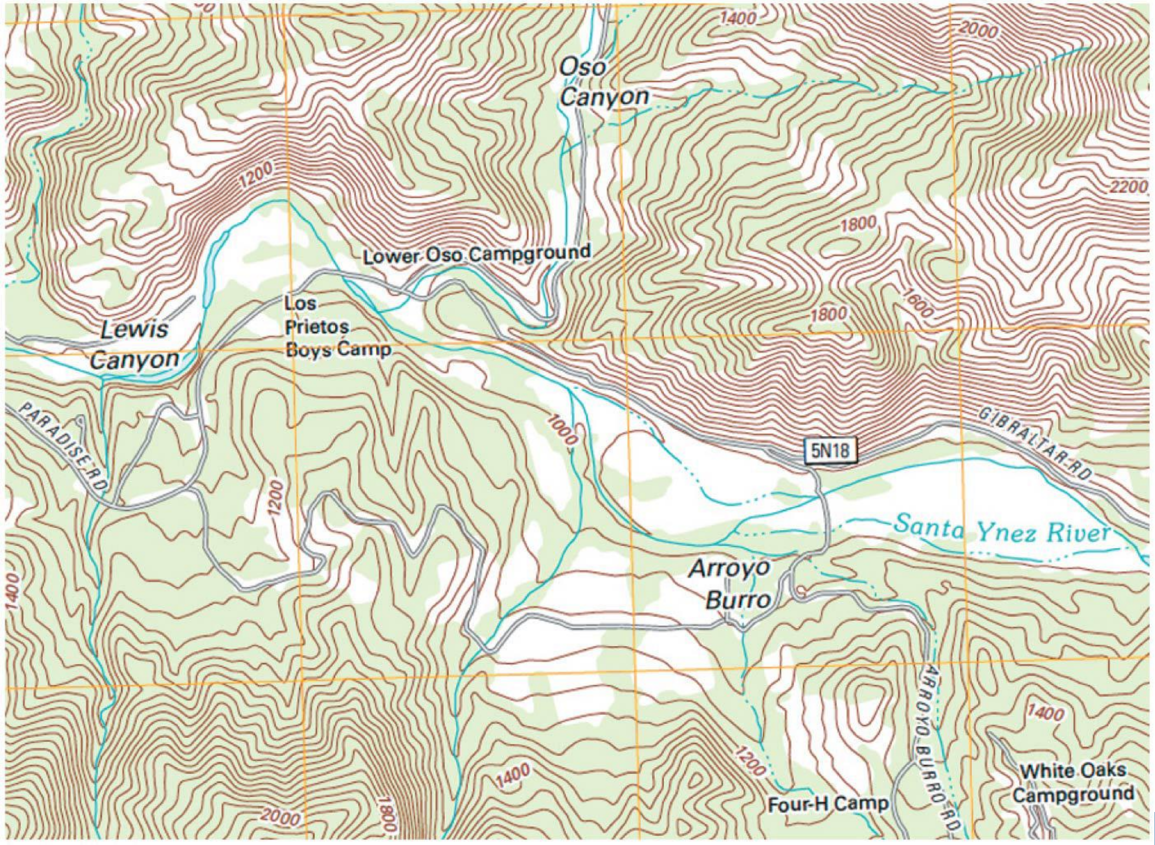
- We'll see how much easier it is to calculate potential vs. the field since we're dealing with scalar quantities.



Equipotential surfaces (analogy)

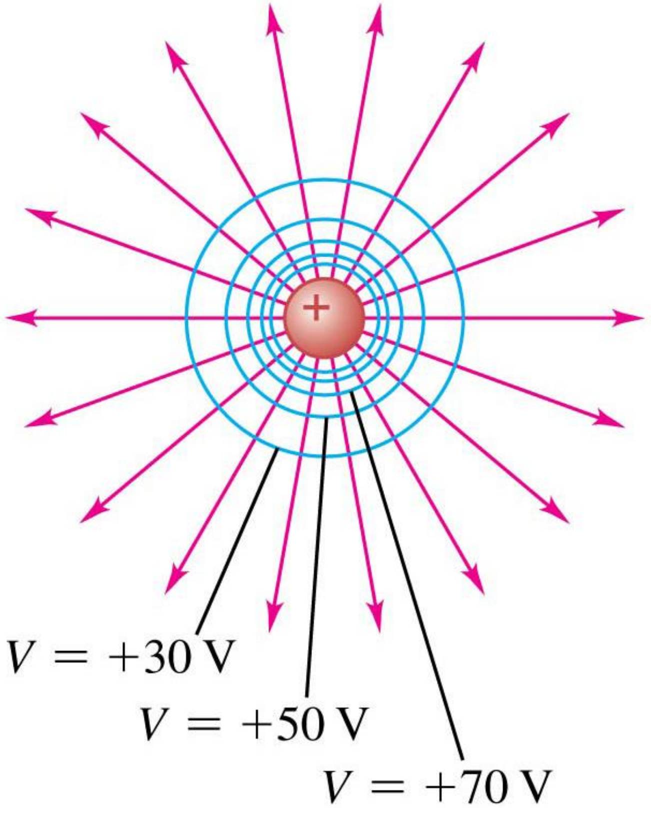
- Electric field lines helped us visualize electric fields. Similarly, potential at various points in a field can be represented by equipotential surfaces.

- The idea comes from contour maps created for hiking. Contour lines indicate the same elevation (same gravitational potential energy).



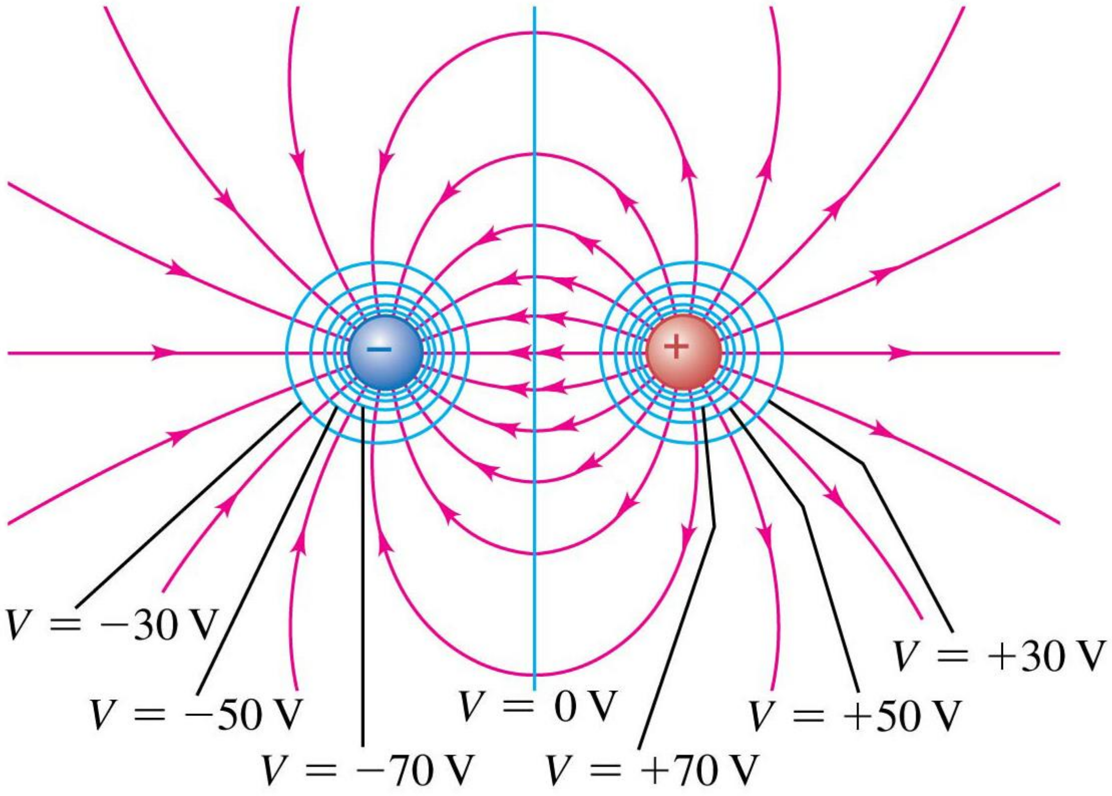
Equipotential surfaces (example)

- An equipotential surface is a surface on which the electric potential is the same at every point.
- Electric field lines and equipotential surfaces are always mutually perpendicular.
- Image shows cross sections of surfaces (blue) and field lines (pink) for a single positive charge.



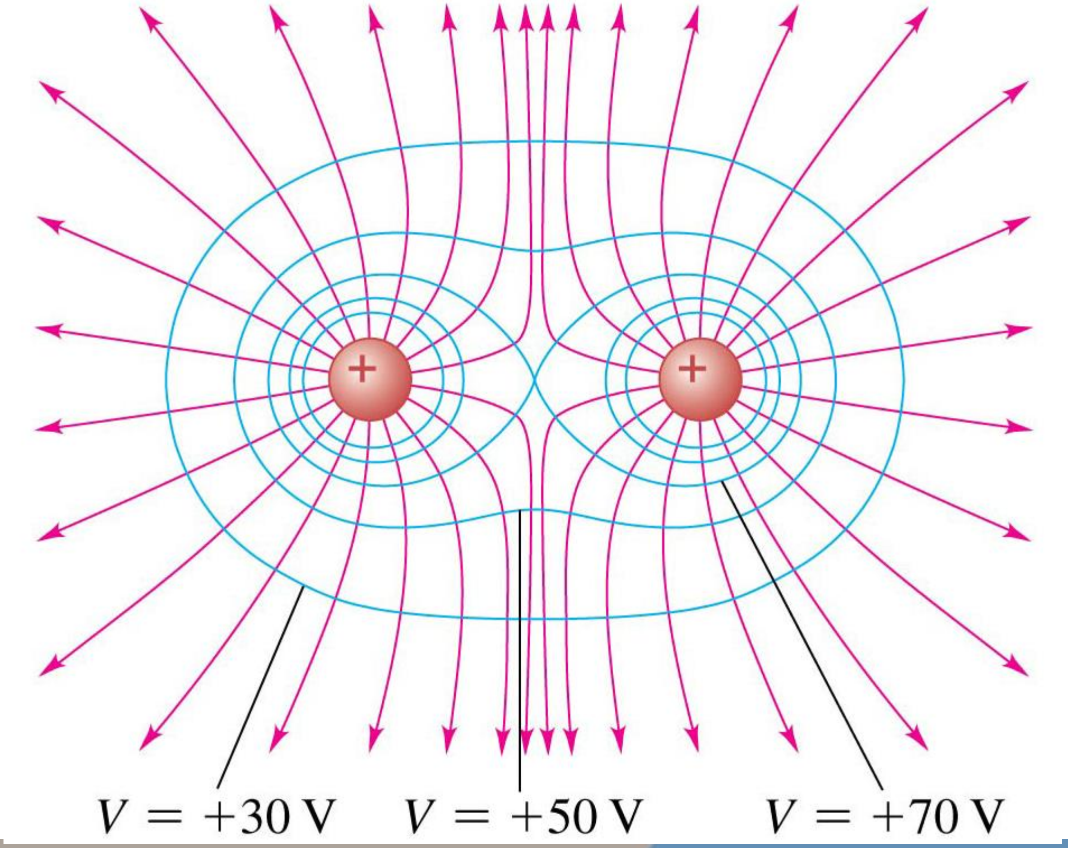
Equipotential surfaces (example)

- For a dipole, both positive surfaces and negative surfaces can be drawn and labeled.



Equipotential surfaces (example)

- The plot for two positive charges shows that even in a region with no E-Field (center) there exists a potential.
- Therefore the electric field magnitude is not necessarily constant over the equipotential surface.



Potential gradient

- If we know the electric field at various points we can find the electric potential through $V_a - V_b = \int_a^b \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}}$.

- We can turn this around; if we know the potential at various points then we can determine the field.

- For rectangular coordinates:

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

- Thus the electric field is the negative gradient of V:

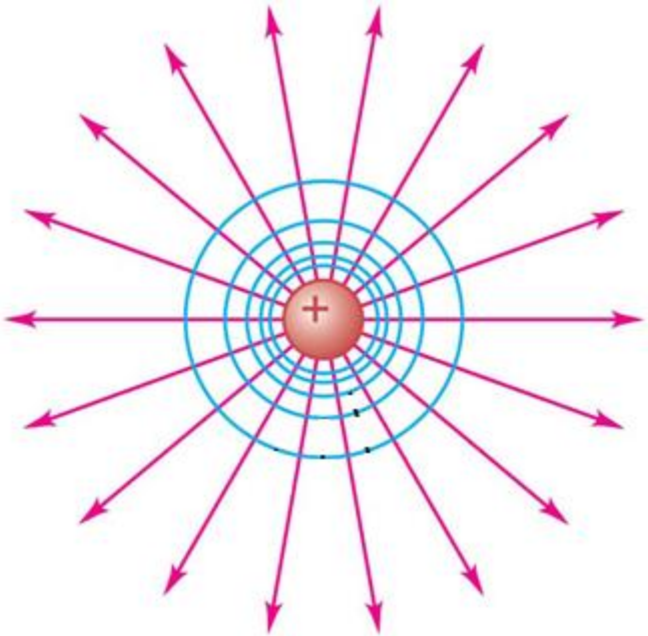
$$\vec{\boldsymbol{E}} = -\vec{\nabla} V$$

Ex. 23.13 – Potential and field of a point

- If the electric field has a radial component with distance r from a point, the **radial electric field** is: $\boxed{E_r = -\frac{\partial V}{\partial r}}$

- For a point charge q, the potential at distance r is given by
$$V = k\frac{q}{r}$$
→ **Find the vector electric field.**

- Both methods: $\vec{\boldsymbol{E}} = \vec{\boldsymbol{r}}E_r$ or if you express r in rectangular coordinates ($r = \sqrt{x^2 + y^2 + z^2}$) and differentiate gives same result. Rectangular coordinates methods much more difficult though.



Van de Graaff Generator

- Developed by Robert J. Van de Graaff in 1929, called an electrostatic generator.

- Simplest explanation:
  – The belt is driven in one direction.
  – When it rubs against the bottom roller, the belt gets a net positive charge which it then brings up to the top roller.
  – The belt then deposits the positive charges on the metal sphere attached near the top rollers.
  – There are other effects such as corona discharge occurring near the two rollers as well.