Chapter 36

- Diffraction

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Learning goals

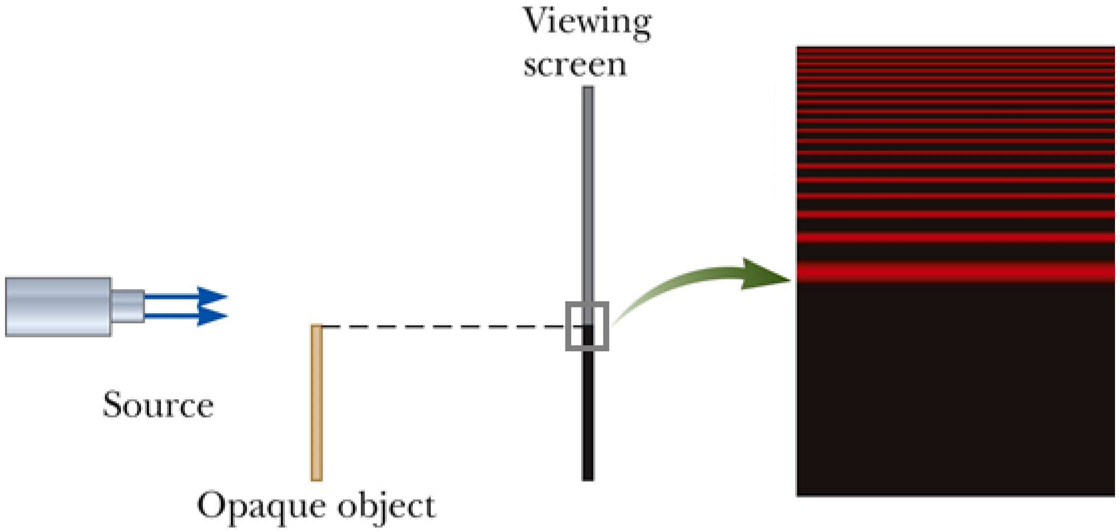
- Fresnel and Fraunhofer diffraction
- Single slit diffraction pattern
- Intensity distribution
  - single slit / double slit
- Diffraction grating
- Other stuff (x-ray diffraction, circular apertures, holography) will not be covered.

Introduction

- Holography is a technique used to record a 3D image onto one or more surfaces.
- Instead of containing a 2D image like a photo, a hologram's surface has an interference pattern on it created by light scattered from the object you're trying to holograph.
- Interference patterns are created due to diffraction of light waves through apertures or around edges of objects.
- In Ch. 35 we saw the pattern for 2 sources, now we extend the discussion to multiple sources.

Fraunhofer diffraction pattern

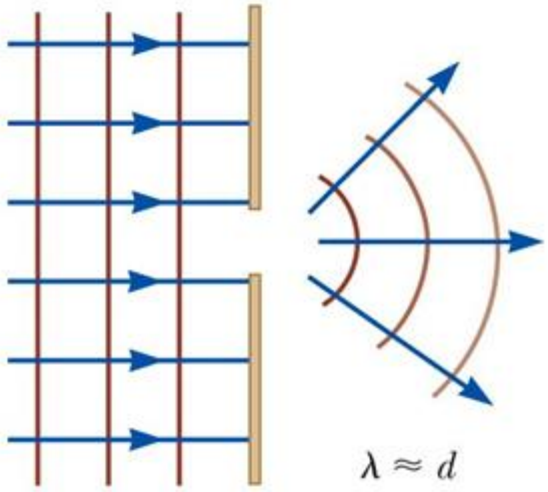
- Geometric optics tells us that when an opaque object is placed between a light source and a screen, the shadow of the object forms a sharp line.
- But if you use a monochromatic source and closely observe the area at the edge of the shadow, you may see some alternating bright and dark fringes creating an interference pattern.
- This is an example of a **Fraunhofer diffraction pattern** where the pattern appears far enough away from the scene of diffraction.
- The diffraction occurring very close to the edge of object is **Fresnel diffraction**.



Diffraction or diffraction pattern?

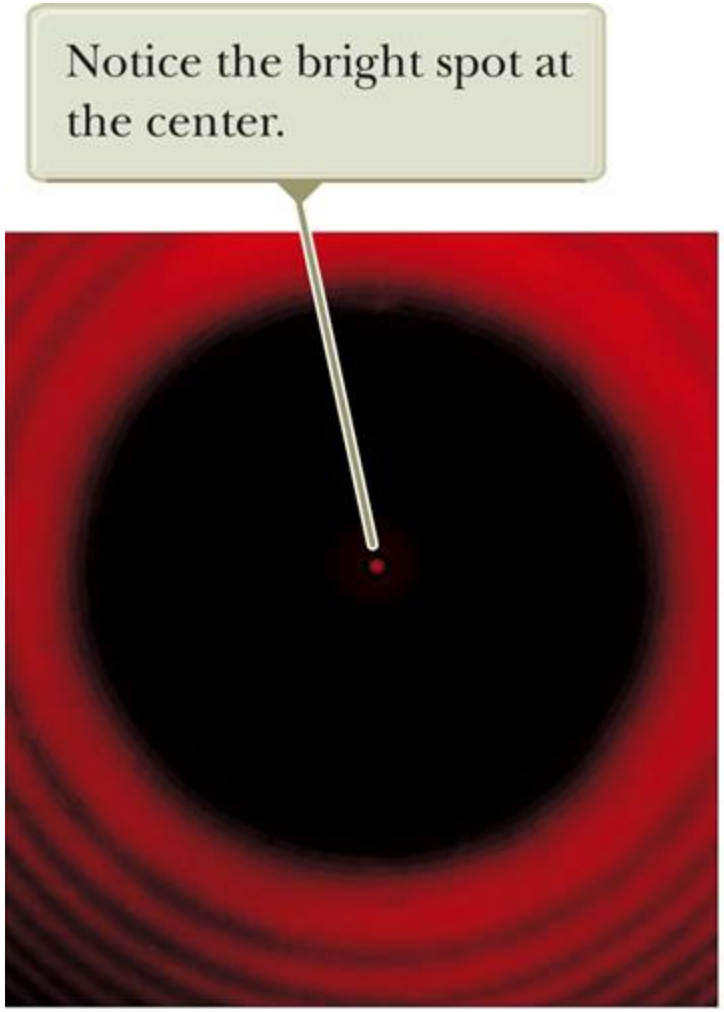
- **Diffraction** is the behaviour of waves spreading out as they pass through a slit. It refers to the bending of wave fronts.

- A **diffraction pattern** is what we've been calling the series of fringes that we see on the screen (it's a misnomer).
  - The correct name is an **interference pattern** because the fringes are caused by interference between parts of the light illuminating difference regions in a slit.



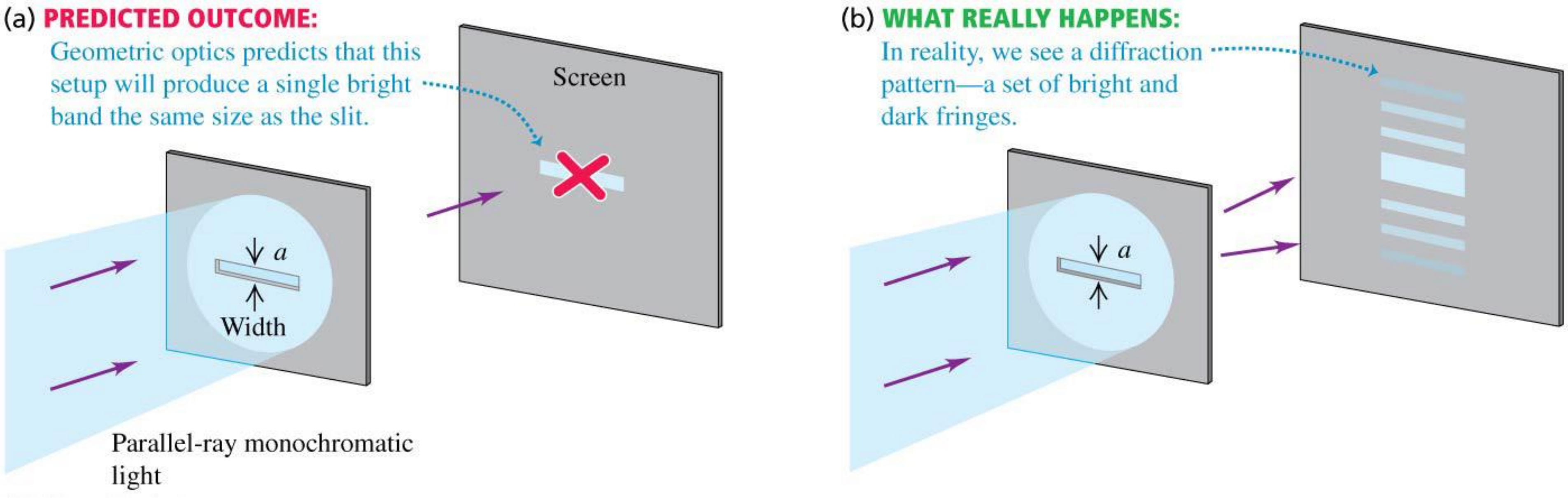
Diffraction around an obstable

- The picture on the right is a diffraction pattern when illuminating a penny.
- The dark circle is the shadow of the penny.
  - Ray optics say that the shadow should be completely dark.
  - Wave optics predicts a bright spot in the center spot.
- This was Poisson's *absurd* spot.
  - (or Arago or Fresnel spot).



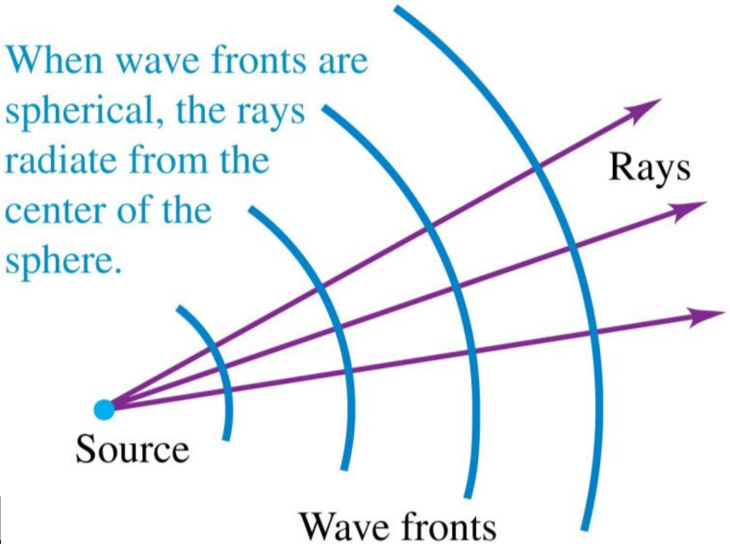
Diffraction from a single slit

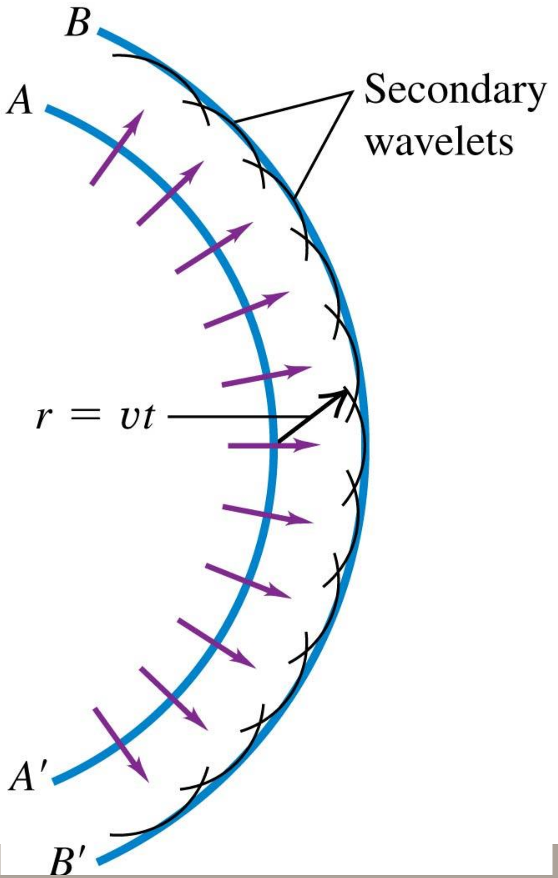
- When monochromatic light is shone on a single thin slit, the beam spreads out vertically and we see a **central maximum**.
  - Above and below we see **side maxima** (secondary).
  - Between the bright fringes are series of dark bands (**minima**).

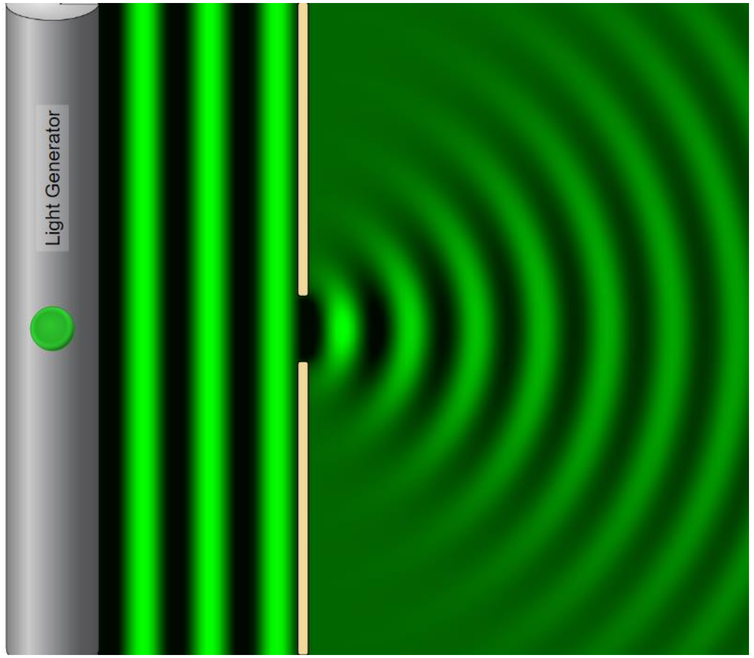


Huygens's principle

- **Huygens's Principle** states that: "every point of a wave front may be considered the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the wave".

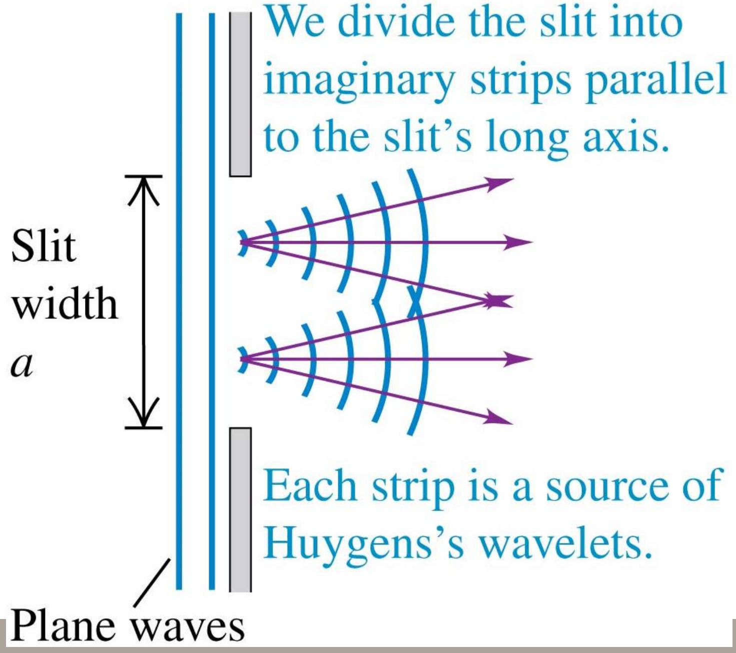


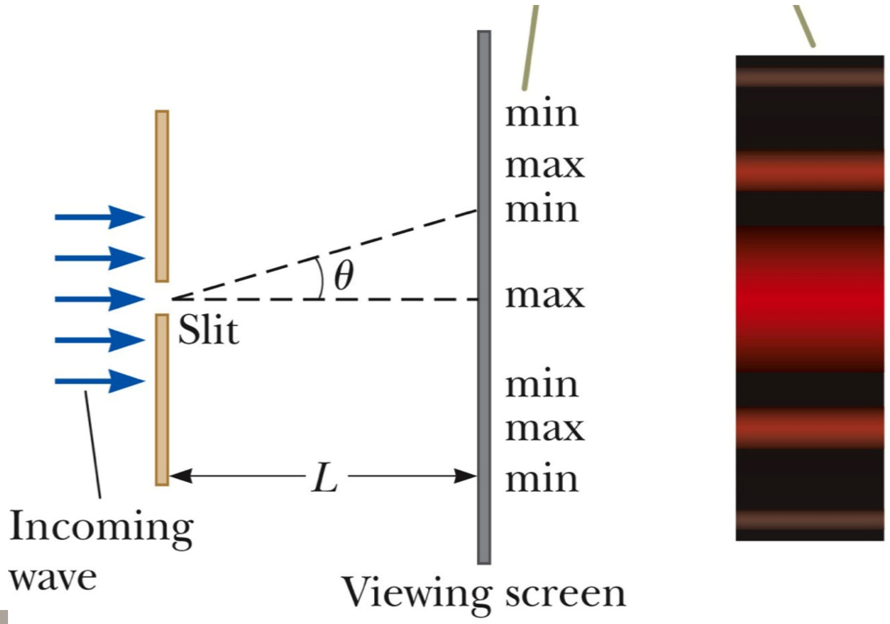




Diffraction from a single slit

- According to **Huygens's principle**, each element of area of the slit opening acts as a secondary source of waves that spread.
- If we have multiple sources of waves that are coming from the single slit then they can interfere!
  - NB. We assume all wave sources are coherent.





Locating the dark fringes ($\delta = \lambda/2$)

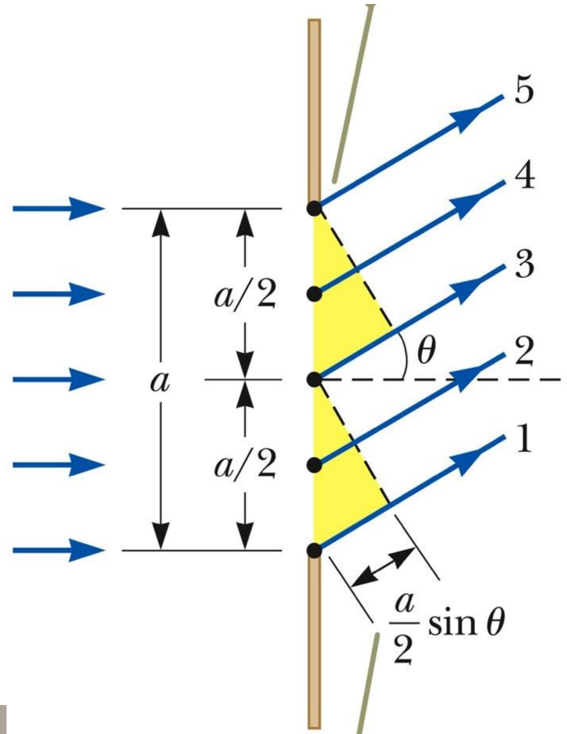
- As an example, let's divide the slit into two halves and assume that there are 5 new light waves.

- Compare waves 1 and 3. Their path difference from the slit to a screen is $\delta = \frac{a}{2}\sin\theta$. (also waves [2 and 4], [3 and 5])

- For $\delta = \frac{\lambda}{2}$, we have **destructive** int.

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2} \quad \rightarrow \quad \boxed{\sin\theta = \frac{\lambda}{a}}$$

- This tells us that for any 2 waves that are separated in the slit by distance $a/2$, their destructive interference condition is $a\sin\theta = \lambda$.



Locating the dark fringes

- We continue the analysis. Now we compare waves 1 and 2 (or 2/3, 3/4, 4/5). The path difference is $\delta = \frac{a}{4}\sin\theta$.

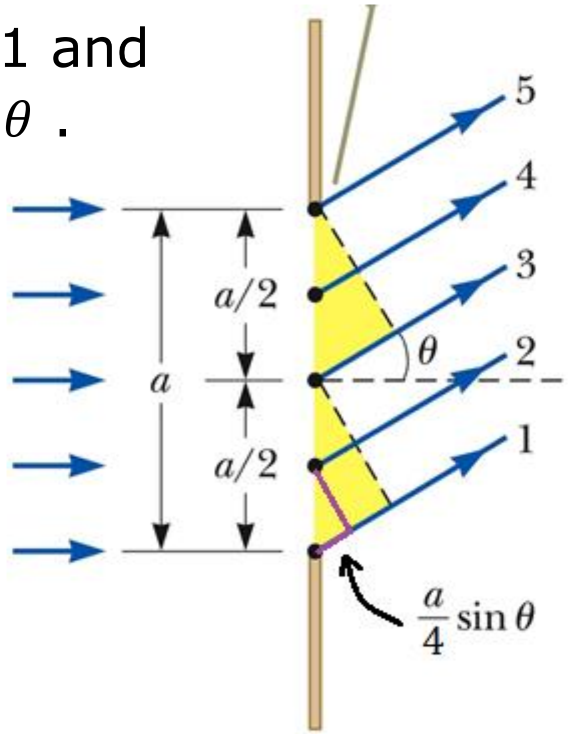
- We apply the same condition as before: $\delta = \frac{\lambda}{2}$

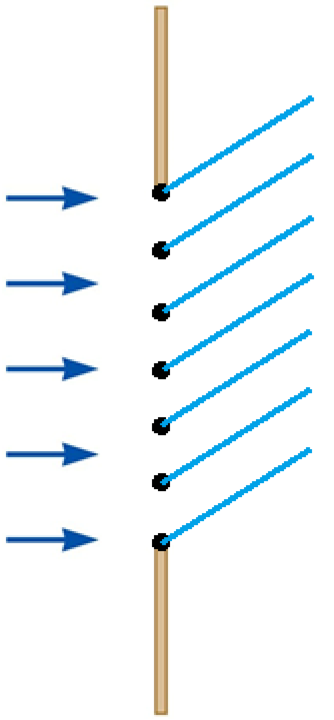
$$\frac{a}{4}\sin\theta = \frac{\lambda}{2} \quad \rightarrow \quad \boxed{\sin\theta = \frac{2\lambda}{a}}$$

- What if we were to divide the slit into more parts? (say 7 waves instead of 5)

$$\frac{a}{6}\sin\theta = \frac{\lambda}{2} \quad \rightarrow \quad \boxed{\sin\theta = \frac{3\lambda}{a}}$$

- Do you see the pattern?





Locating the dark fringes

- In general, destructive interference occurs for a **single slit** of width a if

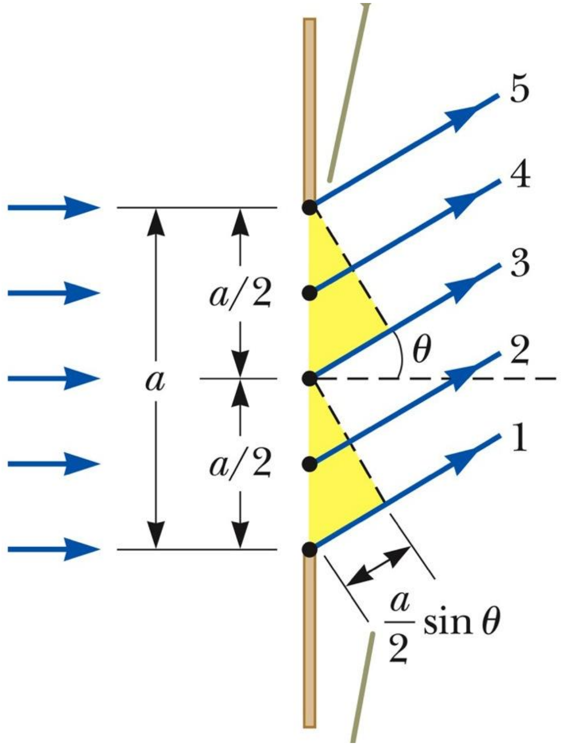
$$\boxed{\sin\theta_{\text{dark}} = \frac{m\lambda}{a}} \qquad \text{for } m = \pm1, \pm2, \pm3 \ldots$$

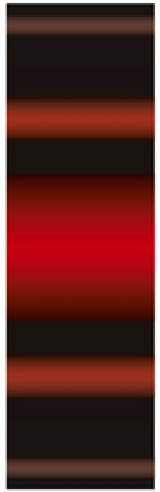
Note. $m = 0$ does not work!

- The distance to the dark fringes is:

$$\boxed{y_{dark} \approx L\left(\frac{m\lambda}{a}\right)} \quad \text{(for small angles)}$$

- The general features of the pattern are:
  - Broad central bright fringes.
  - **Secondary fringes are weaker in brightness**.
  - Bright fringes lie ~halfway between the dark fringes.
  - Since $m \neq 0$, no central dark fringe.





Ex. 36.1 – Single Slit Diffraction

- You pass 633 nm light through a narrow slit and observe the diffraction pattern on a screen 6 m away. The distance on the screen between the centers of the first minima on either side of the central bright fringe is 32 mm. How wide is the slit?

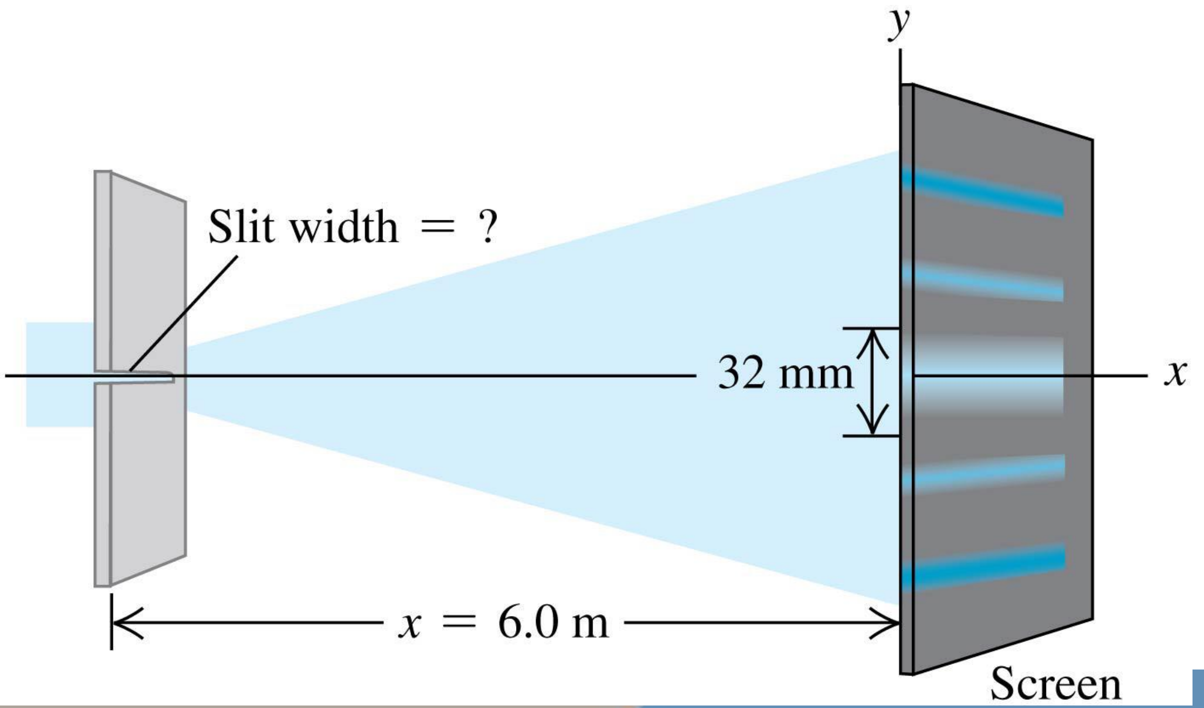
  Solution:
  Like the double slit setup, we can use the small θ approximation:

- $\frac{m\lambda}{a} = \sin\theta \approx \tan\theta = \frac{y}{x}$

  $y_1 = x\frac{m\lambda}{a} = 16 \text{ mm}$

  $a = (6 \text{ m})\frac{(1)(633 \text{ nm})}{16 \text{ mm}}$

  $\boxed{a = 0.24 \text{ mm}}$



Intensity in the single slit pattern

- The intensity of the single slit pattern has distribution:

$$I = I_0 \left[\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}} \right]^2 \qquad \left(\mathbf{NB.}\ \frac{\pi a \sin\theta}{\lambda} \text{ in rads!} \right)$$

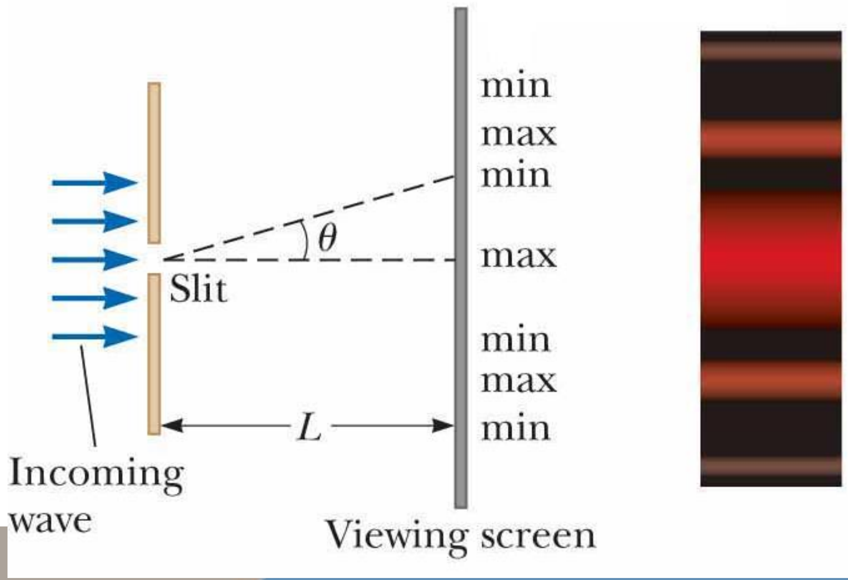
(derivation unnecessary).

- This equation confirms the position of the minima

$$\sin\left(\frac{\pi a \sin\theta}{\lambda}\right) = 0 \quad \rightarrow \frac{\pi a \sin\theta}{\lambda} = m\pi$$

$$\sin\theta = \frac{m\lambda}{a}$$

- Also note, as $\theta \rightarrow 0$, $I \rightarrow I_0$.



Intensity in the single slit pattern

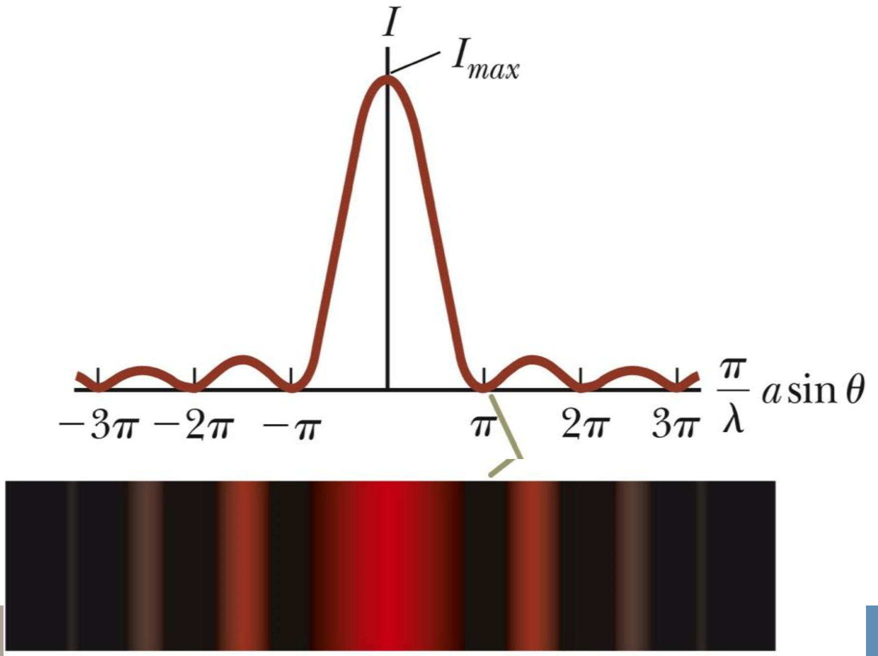
- The figure below shows us the image of the intensity pattern.
  - Most of the light intensity (~85%) is concentrated in the central maximum.

- It is also possible to calculate the light intensity in the side maxima. The drop-off in intensity is rapid.

- The series of intensities are:

$$I_0 \rightarrow 0.0472\ I_0 \rightarrow 0.0165\ I_0 \rightarrow 0.0083\ I_0$$

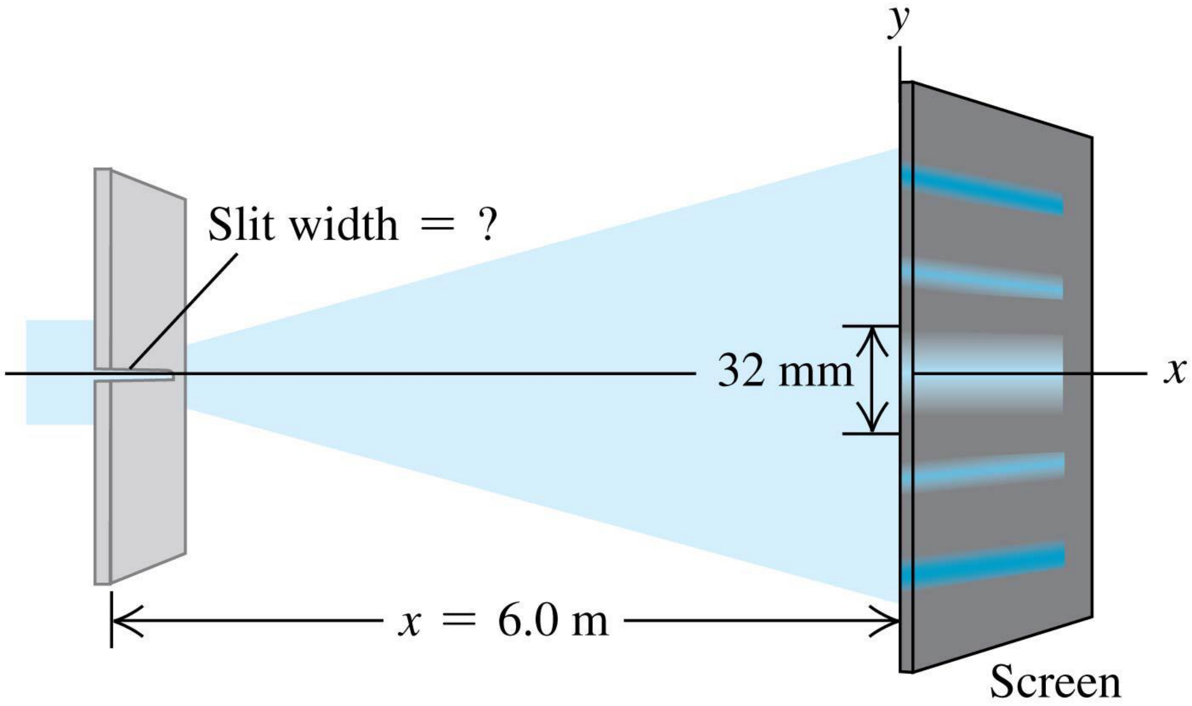
- The first side maximum has less than 5% the intensity of the central.



Ex. 36.3 – Single slit intensity

- In the same setup as Ex. 36.1, the intensity at the center of the pattern is I_0. What is the **intensity** at a point on the screen 3.0 mm from the center of the pattern?

- $\lambda = 633\ \text{nm},\ a = 0.24\ \text{mm}$

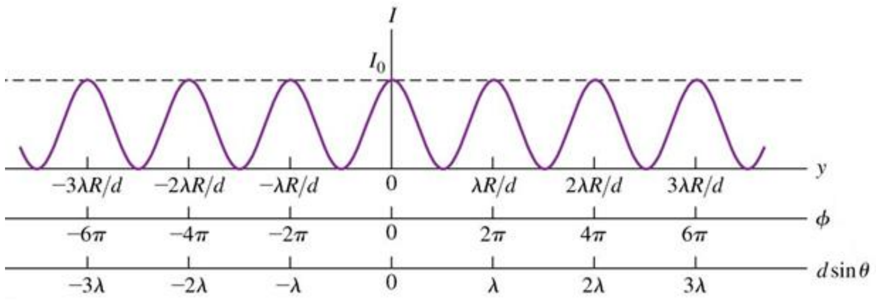


Intensity distribution (2-slit diffraction)

- Since more than one slit is present, we must give consideration to an additional feature.
  - 1) The diffraction pattern due to the individual slits
  - 2) The interference pattern due to the wave coming from multiple slits.

- Recall the 2-slit intensity equation from Ch. 35:

$$I = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$



- We combine this with the single slit pattern to find:

$$I = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)\left[\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}}\right]^2$$

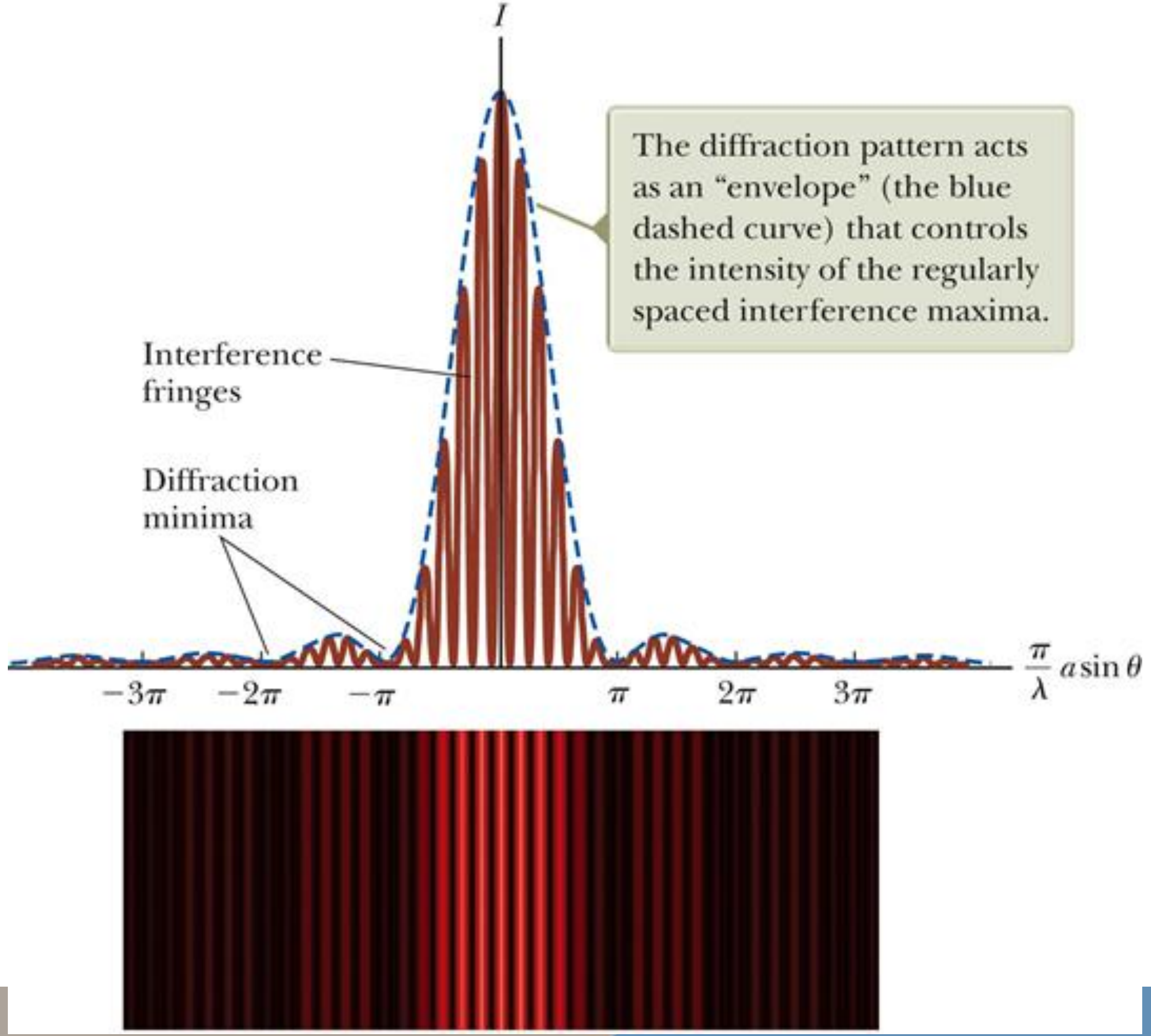
Intensity distribution (2-slit diffraction)

- We have:

$$I = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)\left[\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}}\right]^2$$

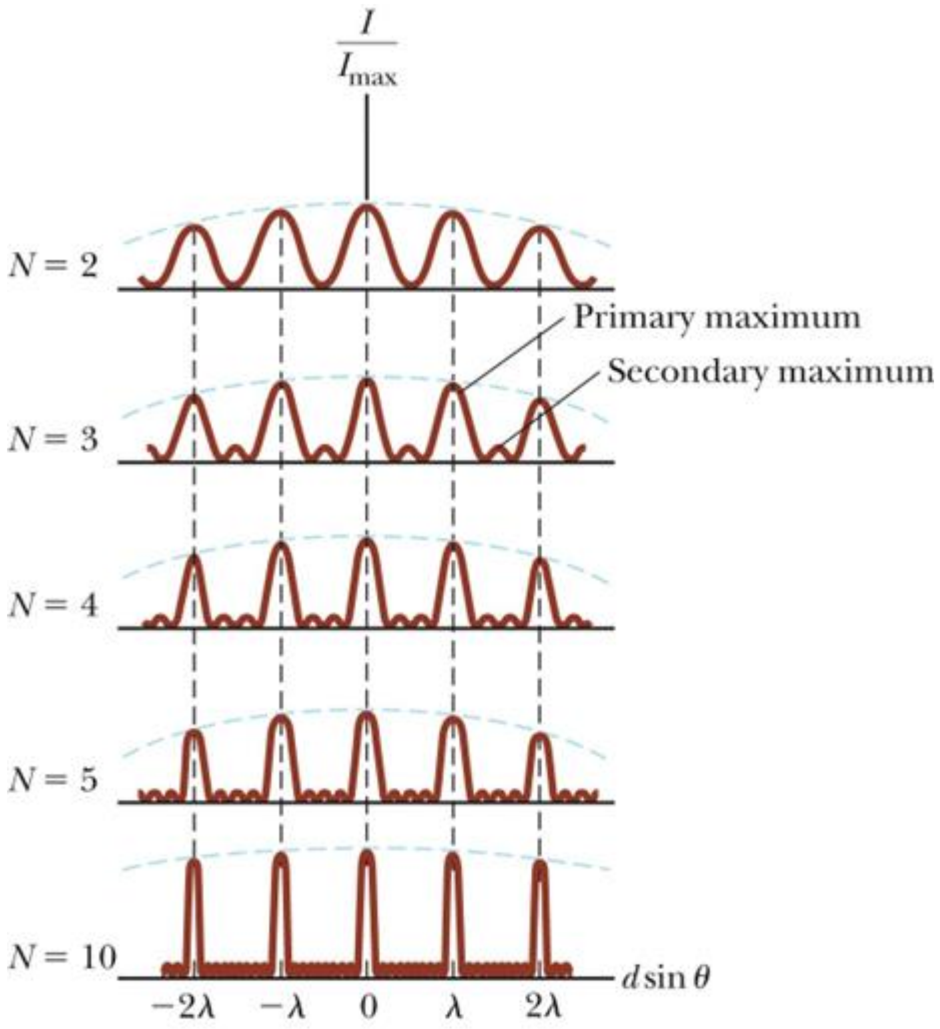
- The square brackets term acts as an envelope (blue dashed line).

- The brown curve shows the $\cos^2$ term which is what we had in chapter 35.



Intensity distribution (many slits)

- As we increase the number of slits, the maxima start to shrink in width but grow in height.
- The height becomes roughly N^2 where N is the number of slits.
- The width becomes roughly $1/N$.
- If we have 100s of slits, all the intensity would be concentrated at the bright spots. This is the case for a **diffraction grating**.



The diffraction grating

- An array of a large number of parallel slits is called a diffraction grating.
- It can be used to analyze light sources such as finding λ.
- A transmission grating is made from cutting parallel grooves on a glass plate to make thousands of slits.
- A reflection grating is made by digging lots of parallel grooves on the surface of a reflective material (like a CD or DVD).

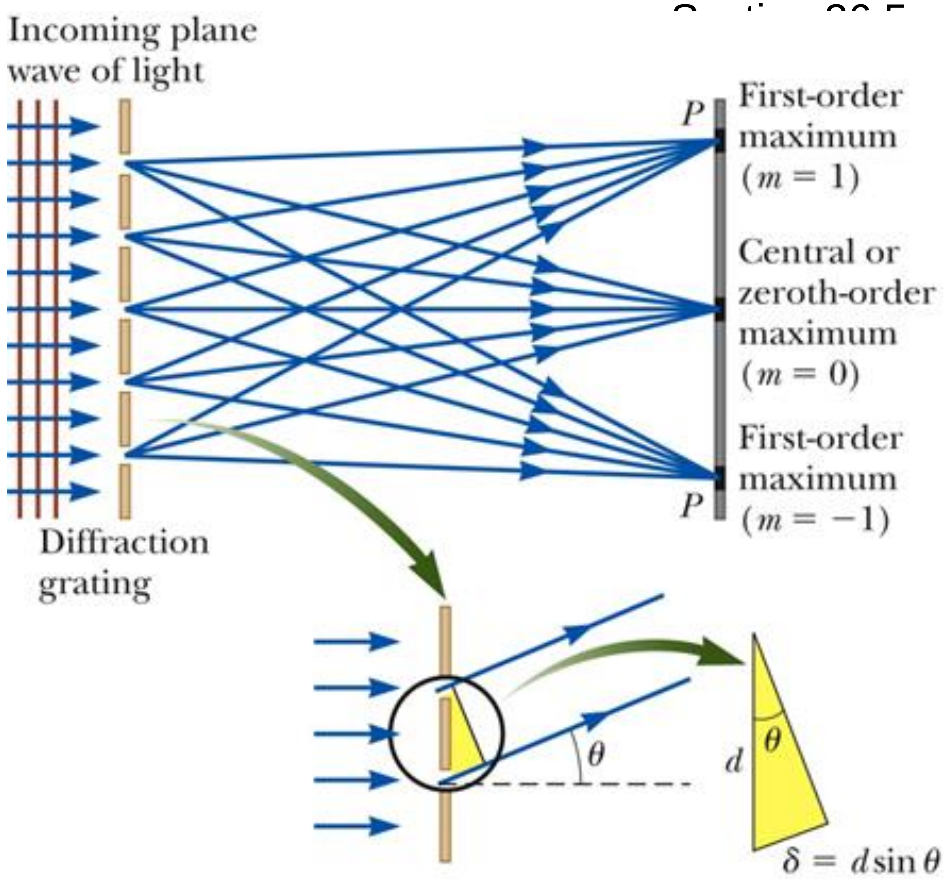
The diffraction grating

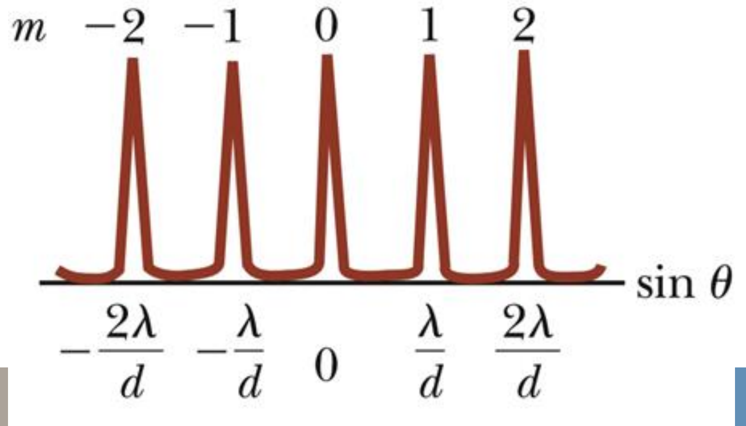
- The equations for constructive interference is the same as for a double slit:

$$\boxed{d\sin\theta_{\text{bright}} = m\lambda} \quad \text{for } m = 0, \pm 1, \pm 2, \ldots$$

- Calculating the separation d between subsequent slits is simple:
  eg. 6000 slits per cm means each slit is 1 / 6000 cm apart.

- If the light source has more than one wavelength then the maxima for different colours will be spatially separated.





Separation of colours using a diffraction grating

- When white light is focused through a grating, the diffraction pattern shows a spectrum of colours. Different colours (wavelengths) constructively interfere at different distances from the center ($\sin\theta_{\mathrm{bright}} = m\lambda/d$)

- Emission spectra of atoms/molecules and white light sources can be imaged.