# Chapter 35

- Interference

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# Learning goals

- Interference of two waves in space.
  - Mechanical waves.
  - Electromagnetic waves (**Young's double slit experiment**).
- **Intensity distribution** of an interference pattern.
- Interference in thin films*
- Measurement of *tiny* distances using interference conditions*

# Introduction

- When observing a soap bubble, you can see a variety of colours and patterns when light shines on it at different angles.
- How is it possible for a colourless object to produce all these cool colours? Similar things happen when you look at reflections from the back of a DVD.
- This chapter presents optical effects (such as interference) that depend on the wave nature of light.
  - These are grouped under **physical optics**.

# Principle of superposition

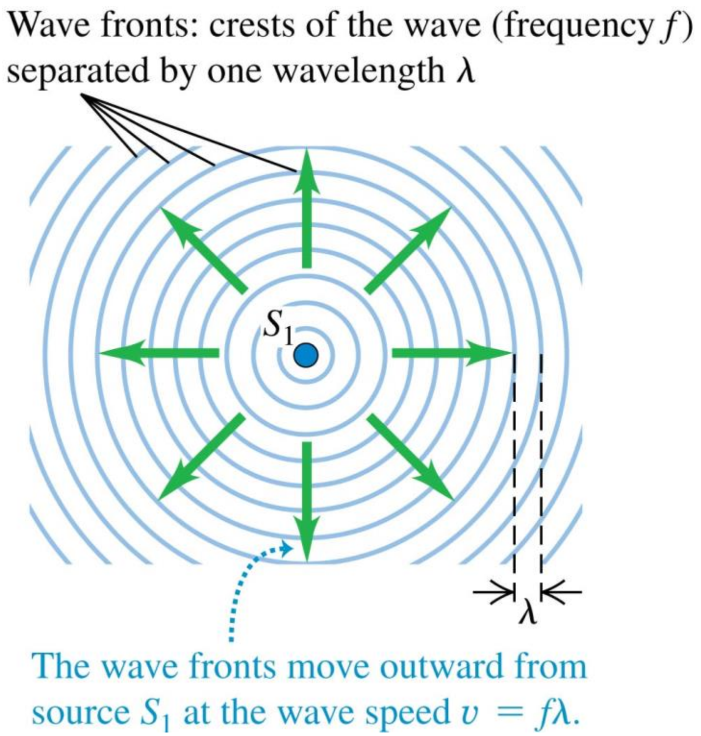
- The term **interference** refers to a situation in which two or more waves overlap in space.
  - Different than general "english" use of the word.

- The waves follow the **principle of superposition**:

  When two or more waves overlap, the resultant displacement at any point and at any instant is found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

# Circular (spherical) wavefronts

- The figure shows a "snapshot" of **sinusoidal** waves with a given wavelength.
- The **speed** of the wave is constant: $v = f\lambda$
- The waves can be 2D (waves on surface of water) or 3D (sound wave or light waves in air).
- Interference is easiest to see when we combine these types of simple sinusoidal waves.



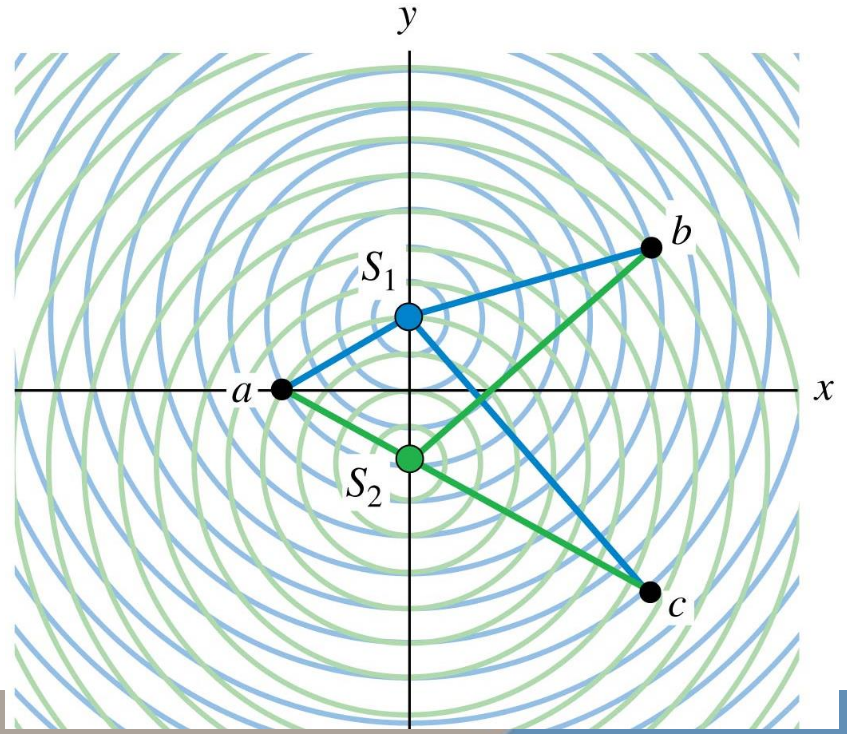

- https://phet.colorado.edu/sims/html/wave-interference/latest/wave-interference_en.html

# Constructive and destructive interference

- Consider two waves with same amplitude, wavelength, speed, and phase (coherent).
  - Eg: two speakers driven by same amplifier, two radio antennas powered by same transmitter, monochromatic light through two slits in a screen, etc…

- Points $a$ and $b$ show points of **constructive** interference.
  - Path difference is an integer multiple of wavelength $\lambda$.

- Point $c$ shows **destructive** interference.
  - Path difference is $\left(m+\frac{1}{2}\right)\lambda$



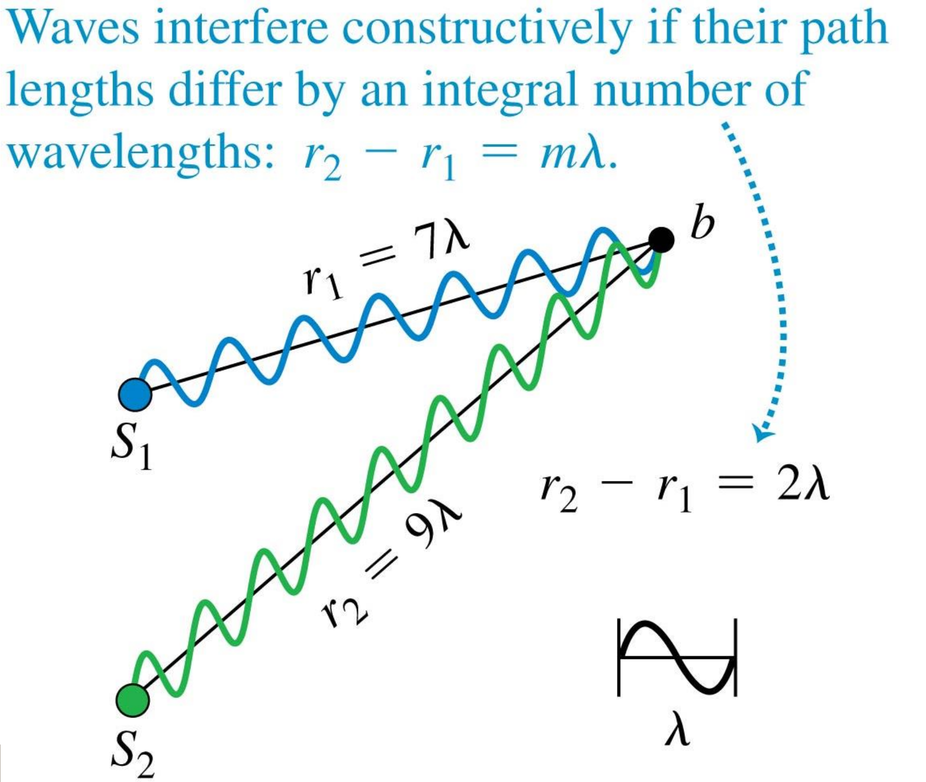
# Conditions for constructive interference

- The distance from each source to the point $b$ is called a **path** and we label them $r_1$ and $r_2$.
  - The difference in distance between the two paths is called the **path difference**: $\Delta r = \delta = r_2 - r_1$ .

- If $\delta$ is equal to an integer-multiple of the wavelength then the two waves arrive "in phase".
  - Results:

    $\boxed{\delta = m\lambda}$, $m = 0, \pm 1, \pm 2, \ldots$

    Constructive interference (more amplitude)



# Conditions for destructive interference

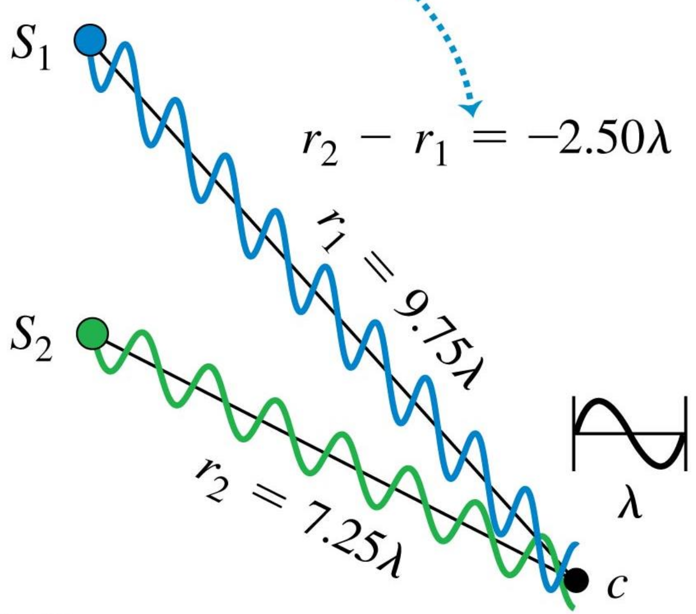
- If the path difference is equal to a half-integer multiple of the wavelength then the two waves arrive $180°$ out of phase
(or "100% out of phase", or "completely out of phase").
  - Results:

$$\delta = \left(m + \frac{1}{2}\right)\lambda, \ m = 0, \pm 1, \pm 2, \ldots$$

Destructive interference (zero amplitude)

- If $\delta$ is equal to anything else then we have partially constructive or destructive interference.

Waves interfere destructively if their path lengths differ by a half-integral number of wavelengths: $r_2 - r_1 = (m + \frac{1}{2})\lambda$.



# Two source interference of water waves

- Two source interference can be seen in water waves (as shown in picture below).

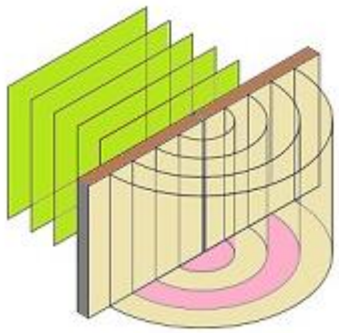
- However, it's not easy to see this result in light since we can't see light traveling in air.

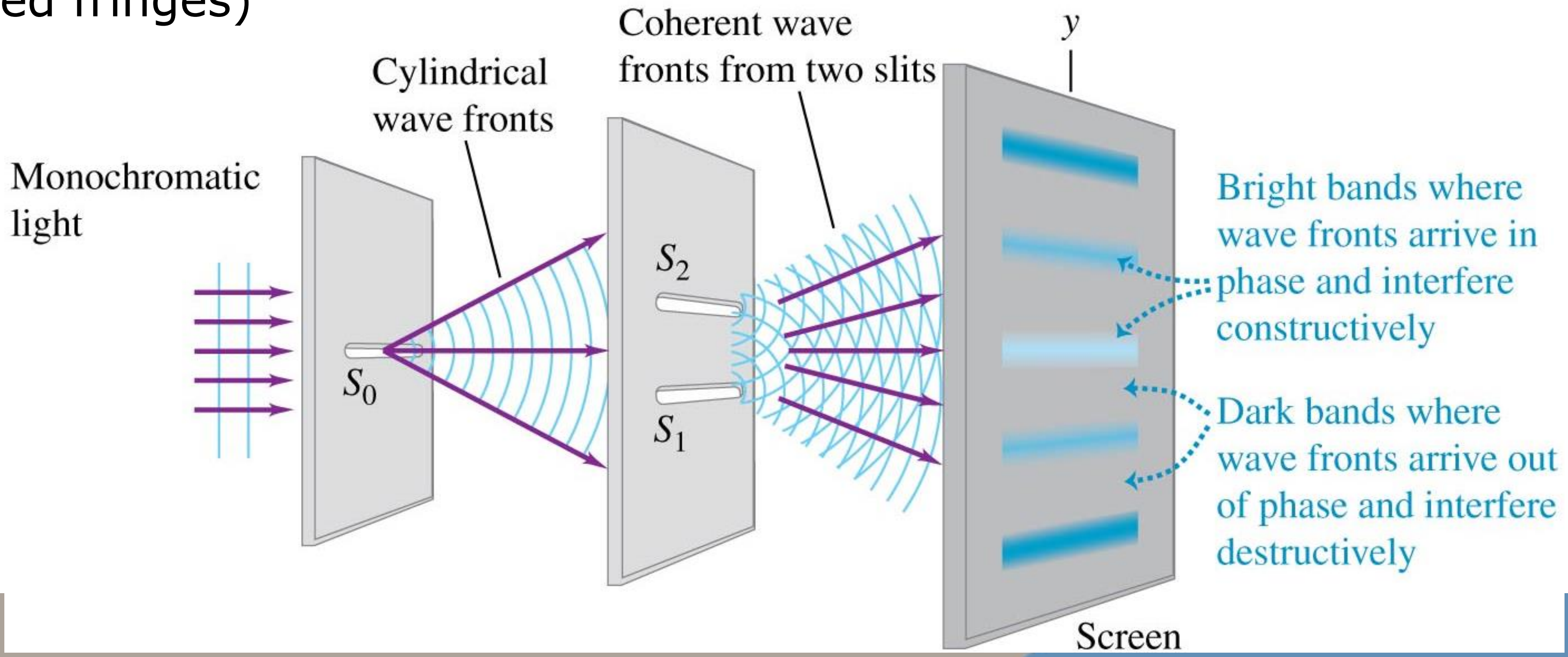
- In 1800, the first demo of two source interference of light waves was shown: **Young's double slit exp.** (next few slides).

# Two source interference of light 1

- Monochromatic light is shone through a single slit to create curved wavefronts then shone through two slits $S1$ and $S2$.

- The interference of waves from the double slit produces a pattern of bright and dark bands (called fringes) on a screen.





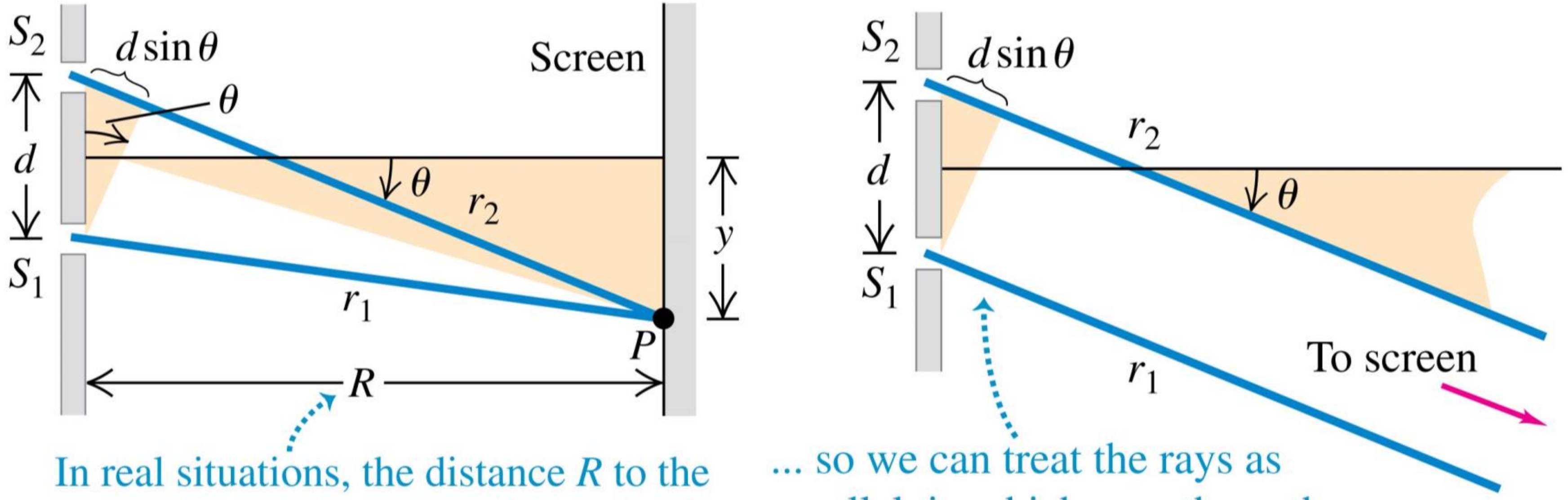
# Two source interference of light 2

- The left figure shows the actual geometry of the path lengths from each slit to the point $P$ on the screen: $\delta = r_2 - r_1$

  The right figure shows the approximate geometry which gives us:

$$\boxed{\delta \approx d \sin\theta}$$

actual geometry




approx. geometry

In real situations, the distance $R$ to the screen is usually very much greater than the distance $d$ between the slits ...

... so we can treat the rays as parallel, in which case the path difference is simply $r_2 - r_1 = d \sin\theta$.

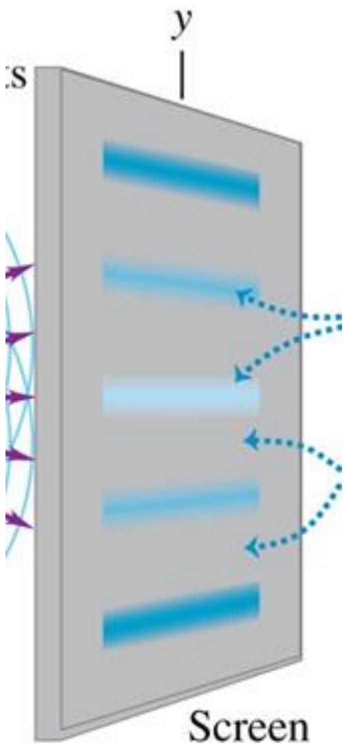
# Double slit interference conditions

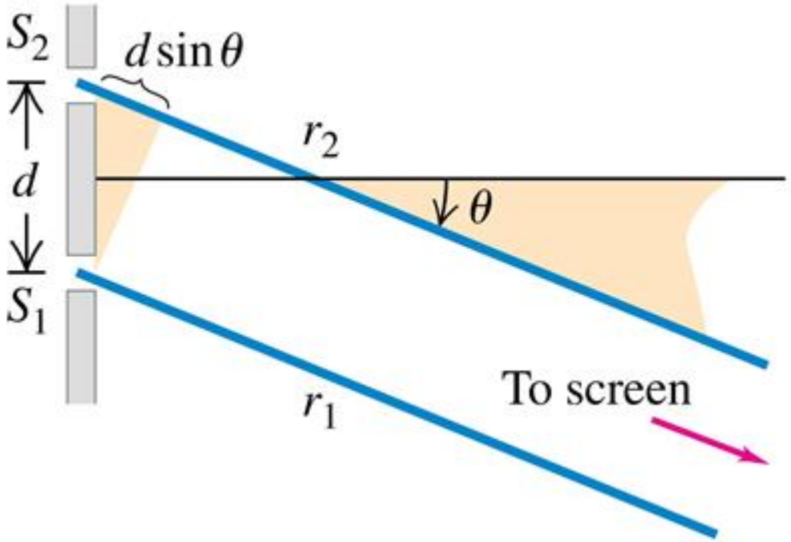
- For constructive interference in the double slit experiment, the path difference is:

$$\delta \approx \boxed{d \sin\theta = m\lambda}$$
$$(m = 0, \pm 1, \pm 2, \dots)$$

- Destructive interference (darkness) occurs for path difference:

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$$
$$(m = 0, \pm 1, \pm 2, \dots)$$



# Distance between fringes on the screen

- Each instance of constructive or destructive interference corresponds to a point on the screen $P$.

- The distance from the central axis to the point $P$ is given by:

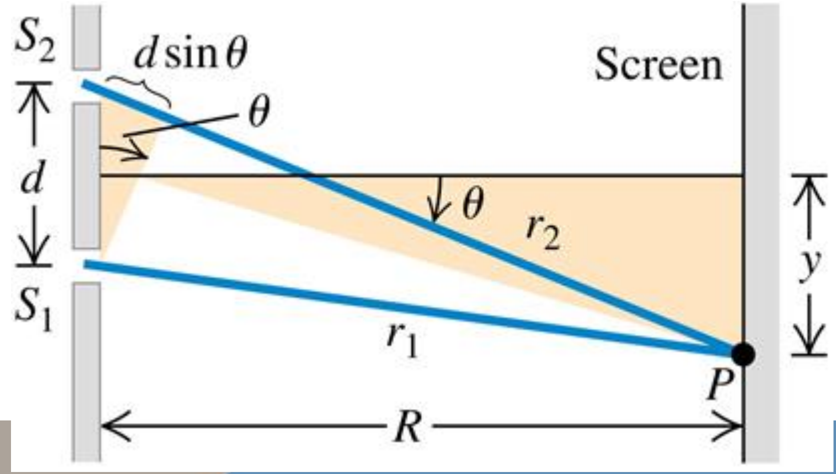
$$y = R\tan\theta$$

which, for small angles (if $\theta < 10°, \tan\theta \approx \sin\theta$), so we have: $\boxed{y = R\sin\theta}$ .

- Specific interference conditions (using $d\sin\theta = m\lambda$):

$$\text{(constructive):} \boxed{y_m = R\frac{m\lambda}{d}}, (m = 0, \pm 1, \pm 2, \dots)$$

$$\text{(destructive):} \boxed{y_m = R\frac{\left(m + \frac{1}{2}\right)\lambda}{d}}, (m = 0, \pm 1, \pm 2, \dots)$$

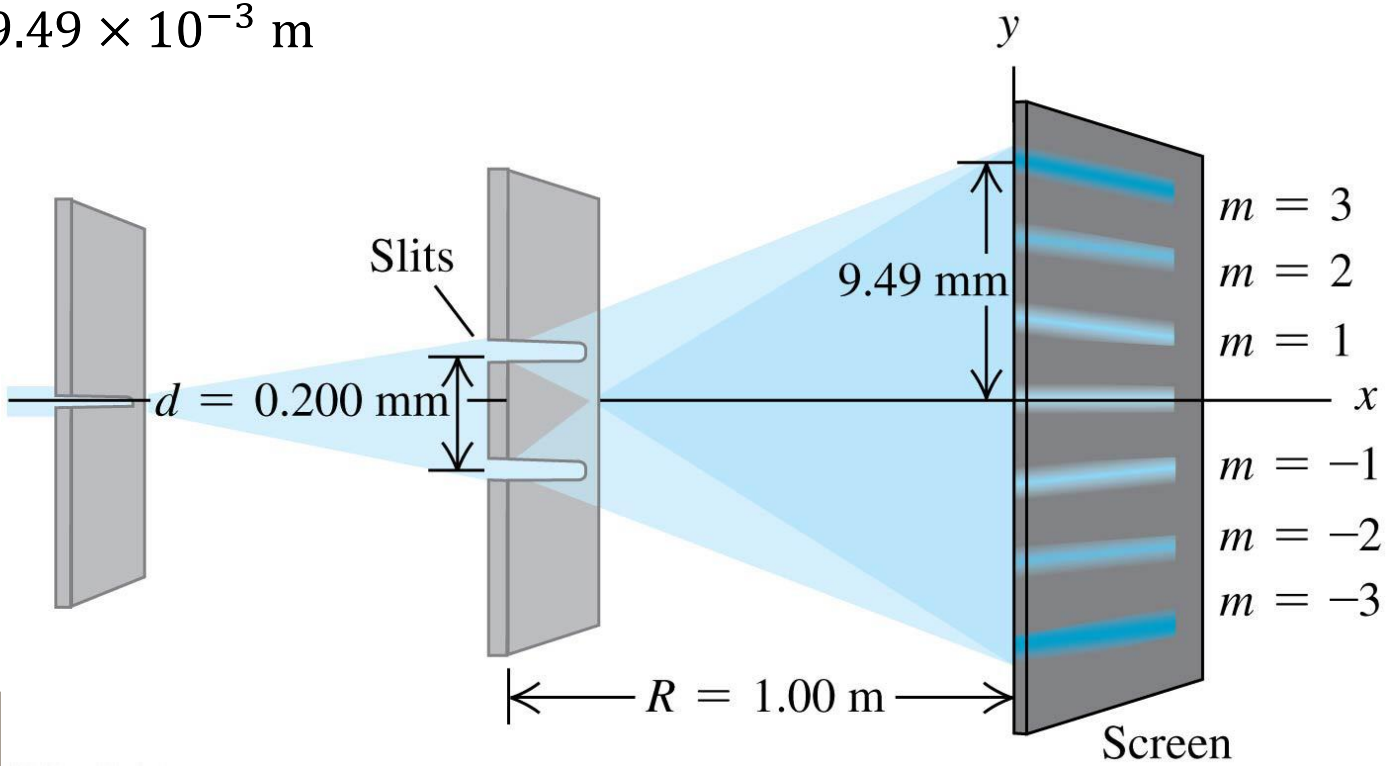


# Ex. 35.1 – Double slit + red light

- The figure shows a double slit exp. with slits 0.200 mm apart and a screen 1.00 m from the slits. The $m = 3$ bright fringe is 9.49 mm from the central fringe.
  - Find the wavelength of light for the given double slit setup:

$$d = 0.2 \times 10^{-3}\ \text{m}, \qquad R = 1\ \text{m}, \qquad y_3 = 9.49 \times 10^{-3}\ \text{m}$$

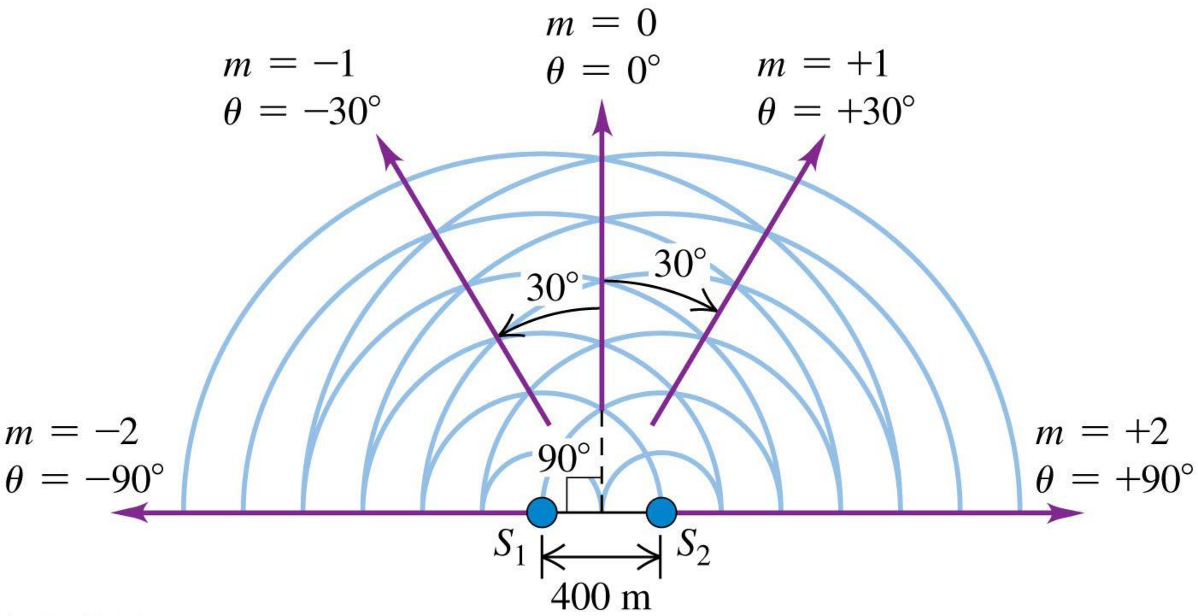
$$y_m = R\frac{m\lambda}{d} \quad \rightarrow \quad \lambda = \frac{d y_m}{mR}$$



# Ex. 35.2 - Radio station broadcast pattern

- A pair of radio antennas that are 400 m apart and operating at 1500 kHz are oscillating in phase. Note: recall that $v = f\lambda$.

- At distances much greater than 400 m, in what directions is the intensity from the two antennas greatest?
  What about the angles for lowest intensity?



# The electric field in interference of light waves

- For interference of mechanical waves at a point, we combine displacement or pressure amplitudes in the medium.

- For electromagnetic waves, we combine amplitudes of the electric and magnetic field vectors. We'll restrict our study to using E-fields.

- The E-field of two waves arriving at a point $P$ with phase difference $\phi$ are:

$$E_1(t) = E\cos(\omega t + \phi)$$
$$E_2(t) = E\cos(\omega t)$$

The field of the combined wave at point $P$ is given by:

$$\boxed{E_P = 2E\left|\cos\frac{\phi}{2}\right|}$$

If $\phi = 0, \pm 2\pi, \pm 4\pi, \text{etc} \ldots$
then $\delta = m\lambda$

If $\phi = \pm\pi, \pm 3\pi, \pm 5\pi, \text{etc} \ldots$
then $\delta = (m + 1/2)\lambda$

# Intensity from electric field

- We can find the **intensity** from the square of the electric field (by definition) since $I \propto E^2$.

- Recall $E_P \propto \cos\phi/2$, therefore the intensity at any point in a two-source interference pattern is given by:

$$\boxed{I = I_0 \cos^2 \frac{\phi}{2}}$$

(note the similarity to Malus's law from Ch. 33).

- The **phase difference** for two source interference comes from the path difference:

$$\boxed{\phi} = \frac{2\pi}{\lambda}(\delta) = \boxed{\frac{2\pi}{\lambda}(d \sin\theta)}$$

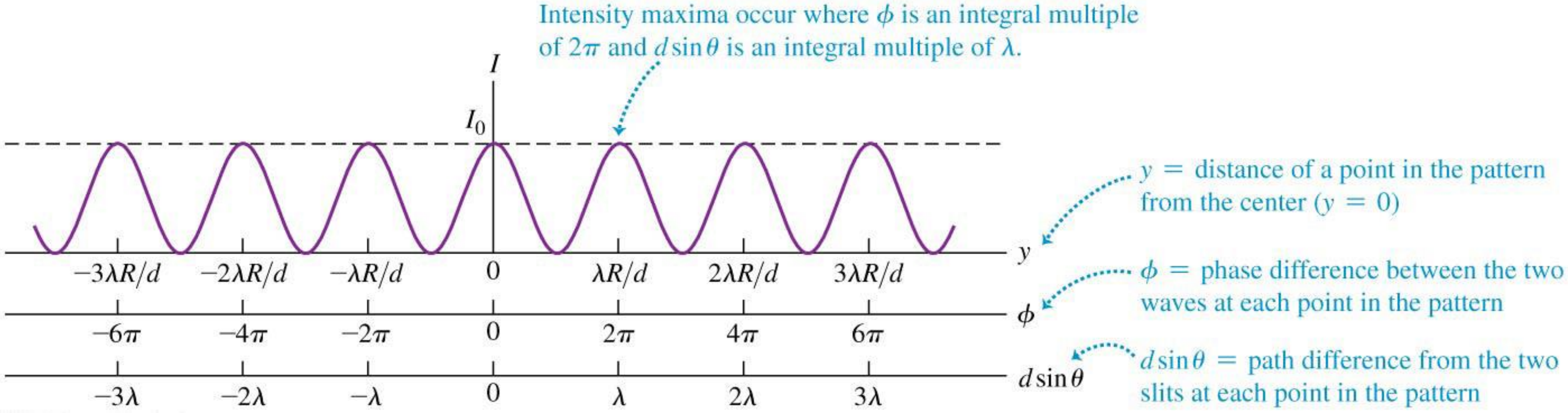
# Intensity distribution of double slit

- In summary we have the equations:

$$I = I_0 \cos^2 \frac{\phi}{2}, \qquad \phi = \frac{2\pi}{\lambda}(d \sin\theta), \qquad \sin\theta = \frac{y}{R} \text{ (if } \theta \text{ small)}$$

- We combine the phase (in terms of double slit conditions) and the intensity to find:

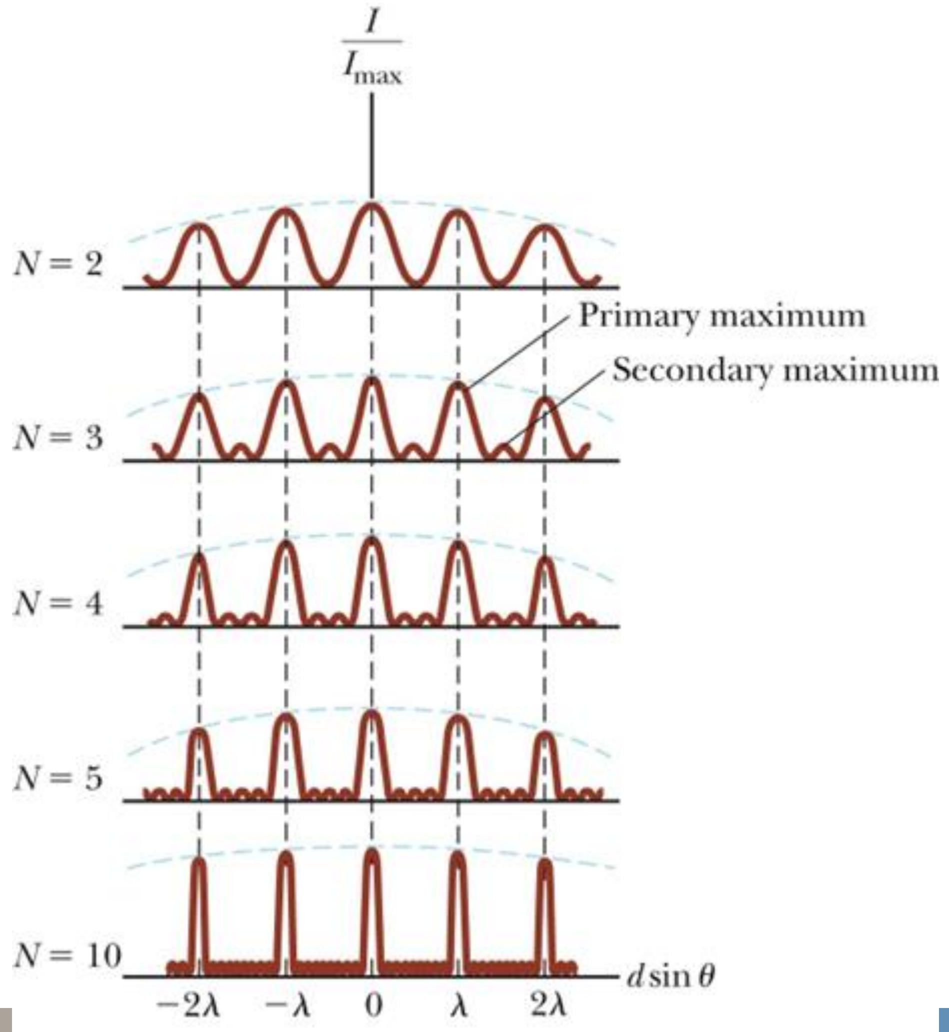
$$\boxed{I = I_0 \cos^2\left(\frac{\pi d y}{\lambda R}\right)} \quad \text{(for small angles)}$$



# Intensity distribution for multiple slits

- If more than two slits are used, we still have a diffraction pattern but it will contain primary and secondary maxima.

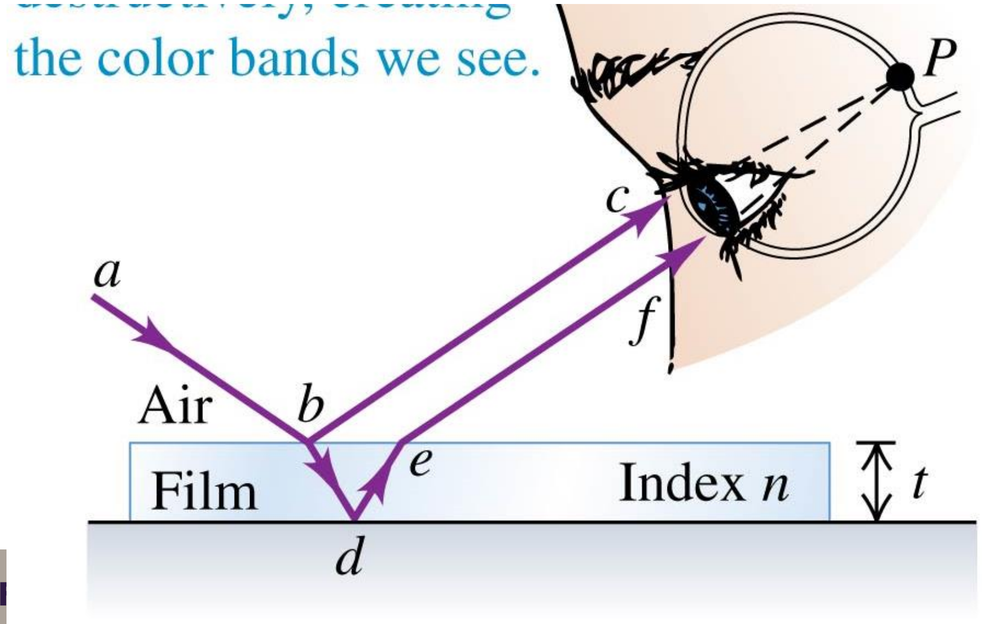
- As more and more slits are used, the secondary maxima reduce in intensity.
  - For $N$ slits, the primary maxima is $N^2$ greater in intensity.
  - The number of secondary maxima is $N - 2$.



# Interference in thin films*

- A common occurrence of interference effects is observed in thin films such as soap bubbles or oil on water.

- The various colours observed when white light is incident on such films results in interference of waves reflected from the two surfaces. The colours depend on the thickness of film and the viewing angle.




# The Michelson interferometer*

- The interferometer was invented in the late 1800s by A. A. Michelson and was used in the famous Michelson-Morley experiment to measure the "ether".
- The device can be used to measure wavelength of light or other very small lengths with high precision/accuracy.
- Where a double slit sends part of a light wave through one slit and part through the other, the Michelson interferometer splits a light wave into two parts using a *beam splitter* then recombines the two waves after they travel a given distance.

# Michelson interferometer, schematic*

- A ray of light is split into two rays by mirror $M_0$ (at $45°$ to the incident beam). This mirror is called the "beam splitter".
- The reflected ray goes towards mirror $M_1$.
- Transmitted towards mirror $M_2$.
- Two paths: $L_1$ and $L_2$.
- Rays recombine at $M_0$ and form an interference pattern.