Chapter 34

- Geometric optics

Michael Wong – PHY 1122 Spring 2023

Learning goals

- Image formation
  - Plane, concave, and convex **mirrors**
  - Refraction
  - Converging and diverging **lenses**

- Spherical and chromatic aberrations*

- Human vision*

- Cameras, microscopes, and telescopes*

Introduction

- A surgeon wears special magnifying glasses so she can get a good view of the surgical site.
- In this chapter we'll see how optics such as lenses and mirrors can be used to create an image.
- Treating light as a ray allows us to understand the principles of image formation.
- Applications include cameras, the eye, microscopes, and telescopes.

Reflection at a plane surface

- Light rays from the object at point P are reflected off a **plane mirror**.

- The reflected rays entering your eye appear to come from the point P' behind the mirror.

- **Note**: in order to see the object point and the image point, there must be no ”barrier” between your eyes and the object/image.

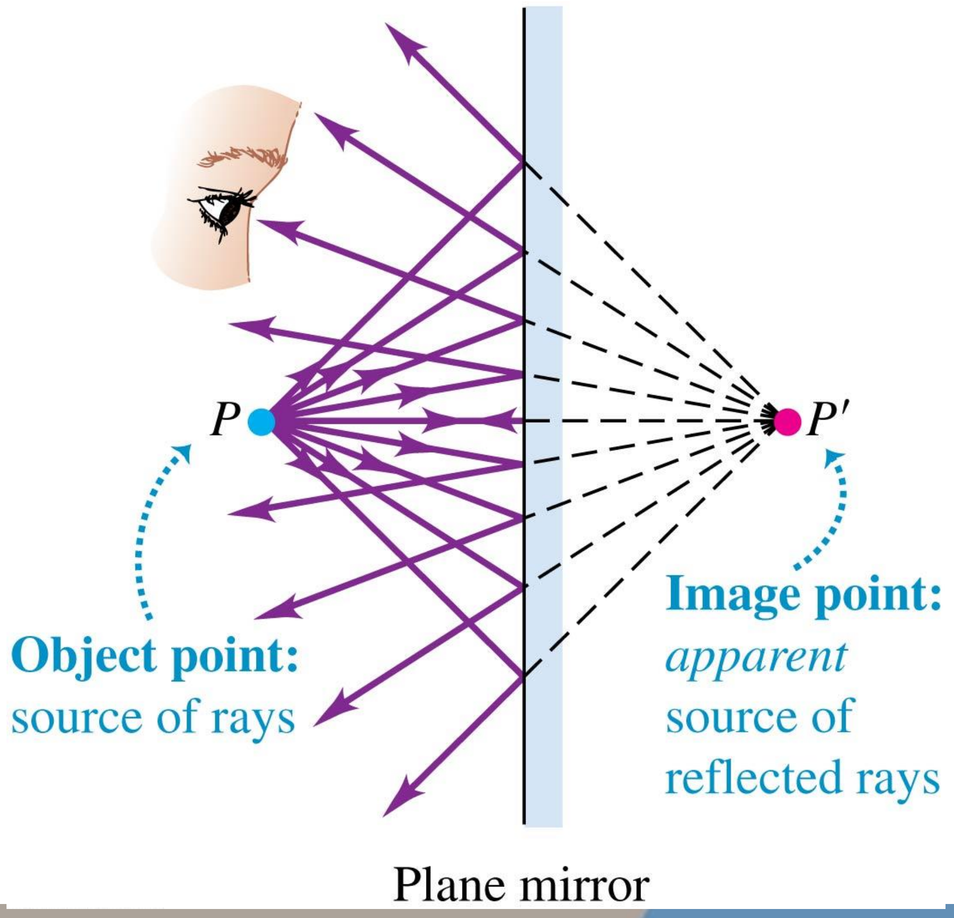


Image formation by a plane mirror

- The ray diagram for the image formed at point P is simple.

- Two light rays are needed to determine the point and we only need to use the law of reflection.

- The mirror forms a **virtual image** at point P'.

- The object distance s and image distance s' are equal.

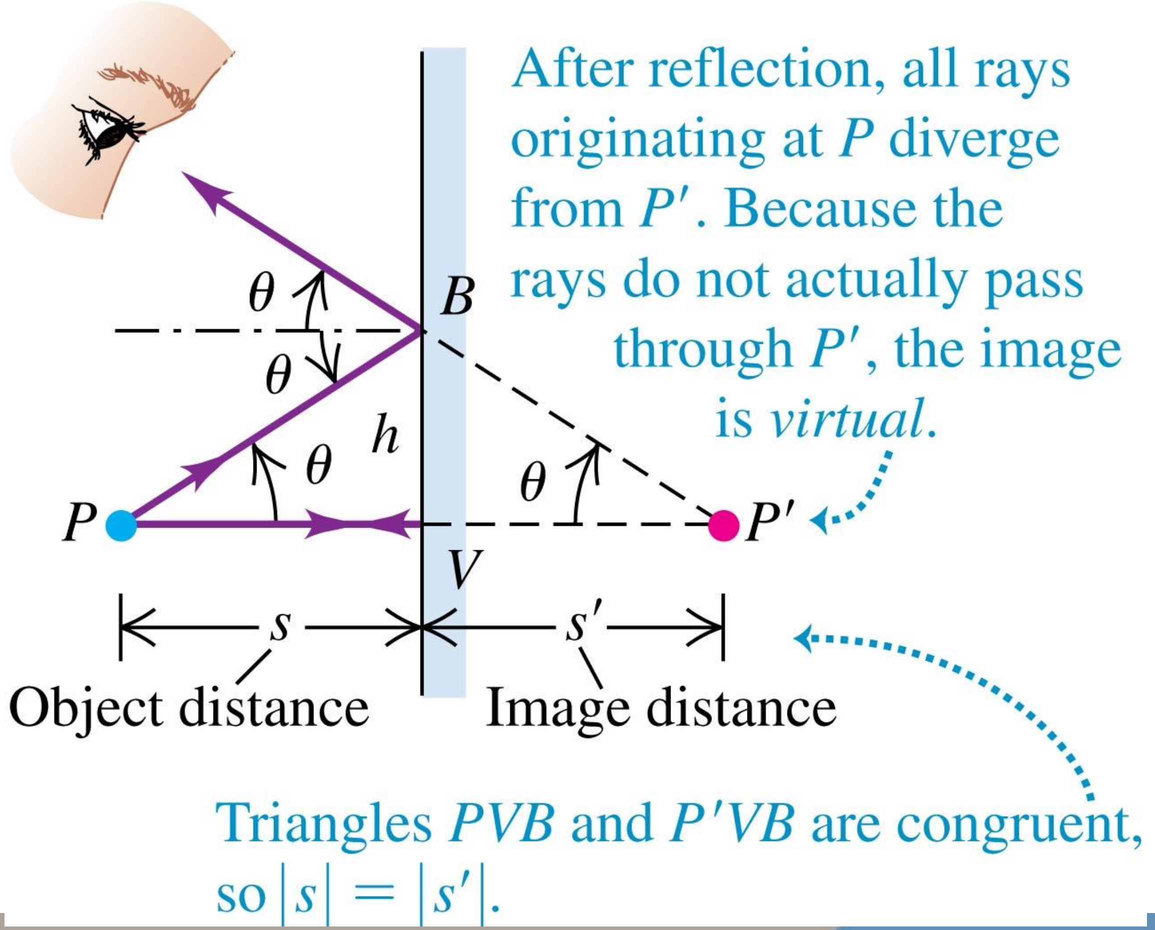
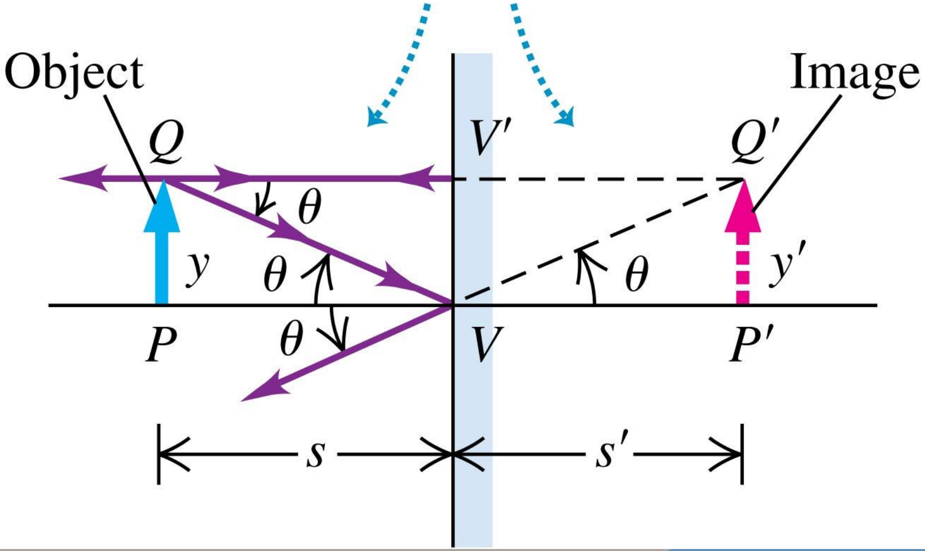


Image formation by a plane mirror

- For an extended object, the image in a plane mirror is also very simple.
- Again, only two light rays are needed to determine the image by reflection.
- The size of the image is equal to the size of the object.
- The **magnification** is $\boxed{M = \frac{y'}{y}} = -\frac{s'}{s}$.
- Magnification is related to s and s' because the triangles are similar.

For a plane mirror, PQV and $P'Q'V$ are congruent, so $y = y'$ and the object and image are the same size (the lateral magnification is 1).

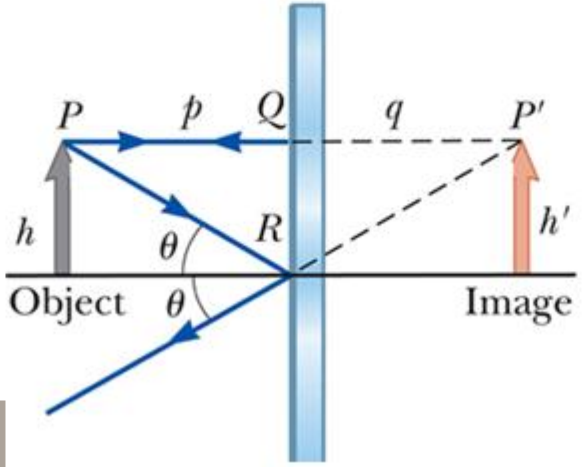


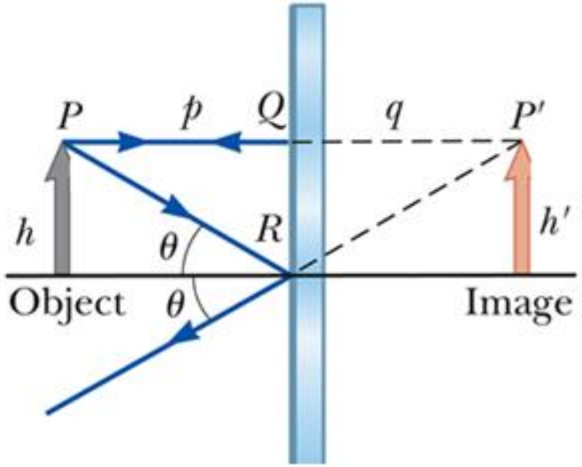
Reversals in plane mirrors

- A plane mirror produces an image that has *apparent* left-right reversal (raise right hand, image raises left hand).
- But why does it reverse left-right but not up-down?
  - Why is my head still up and my feet still down?
- The plane-mirror reversal is actually a **front-back reversal**.

incorrect

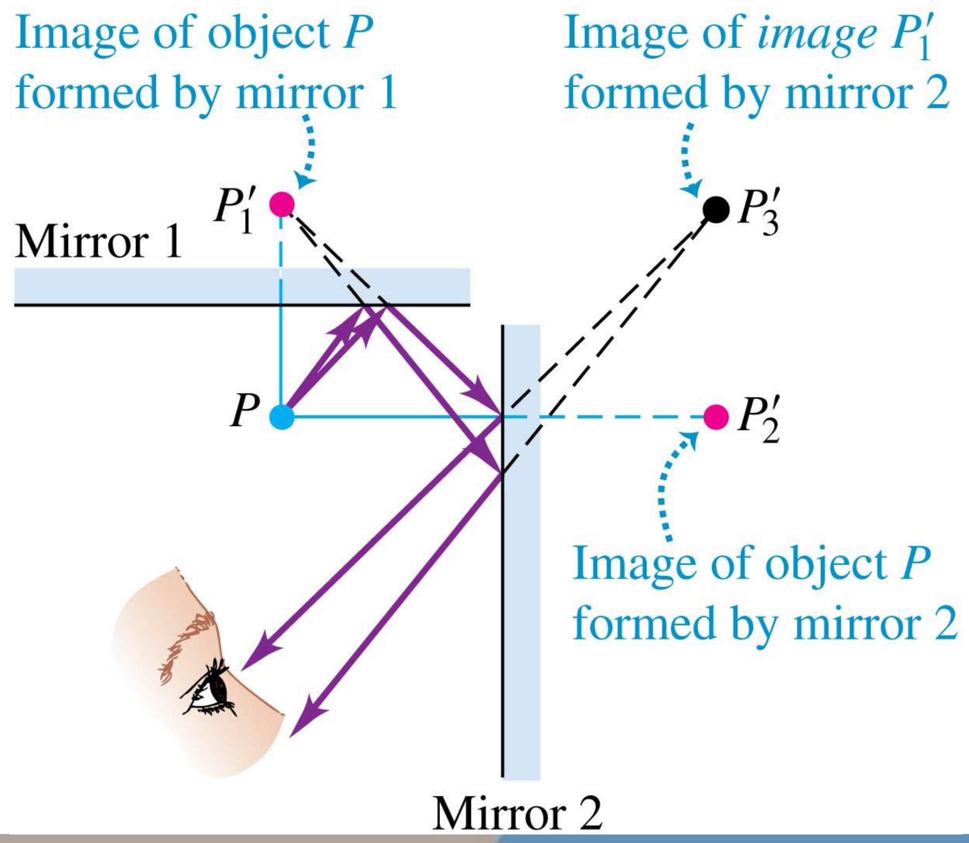
correct





Multiple images formed by two mirrors

- For two perpendicular mirrors, an object at point P can form images at P_1' due to mirror 1 and P_2' due to mirror 2.
- Additionally, an image formed by one mirror can act as an object for the other mirror such as the image formed at P_3'.
- If you have three mirrors at right angles to each other you can have 7 images formed.
  - Note: some images are reversed and some are not!



Spherical mirror with a point object

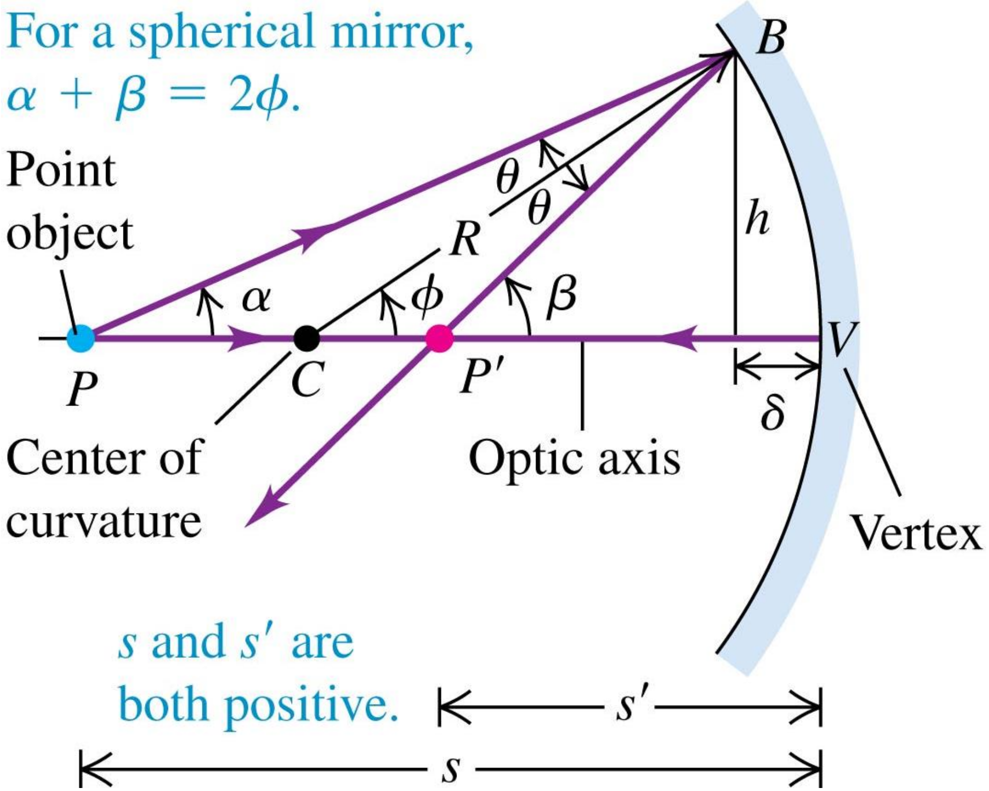
- A **spherical *concave* mirror** with radius of curvature R forms a **real image** P' of the object P.

- s and s' are the object and image distances.

- The mirror equation in terms of s, s', and R is given by:

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}}.$$

- Note that $s > R$ in this example.



Focal point and focal length

- Incoming parallel rays will converge at a point F which has distance f equal to half the radius of curvature: $f = R/2$.

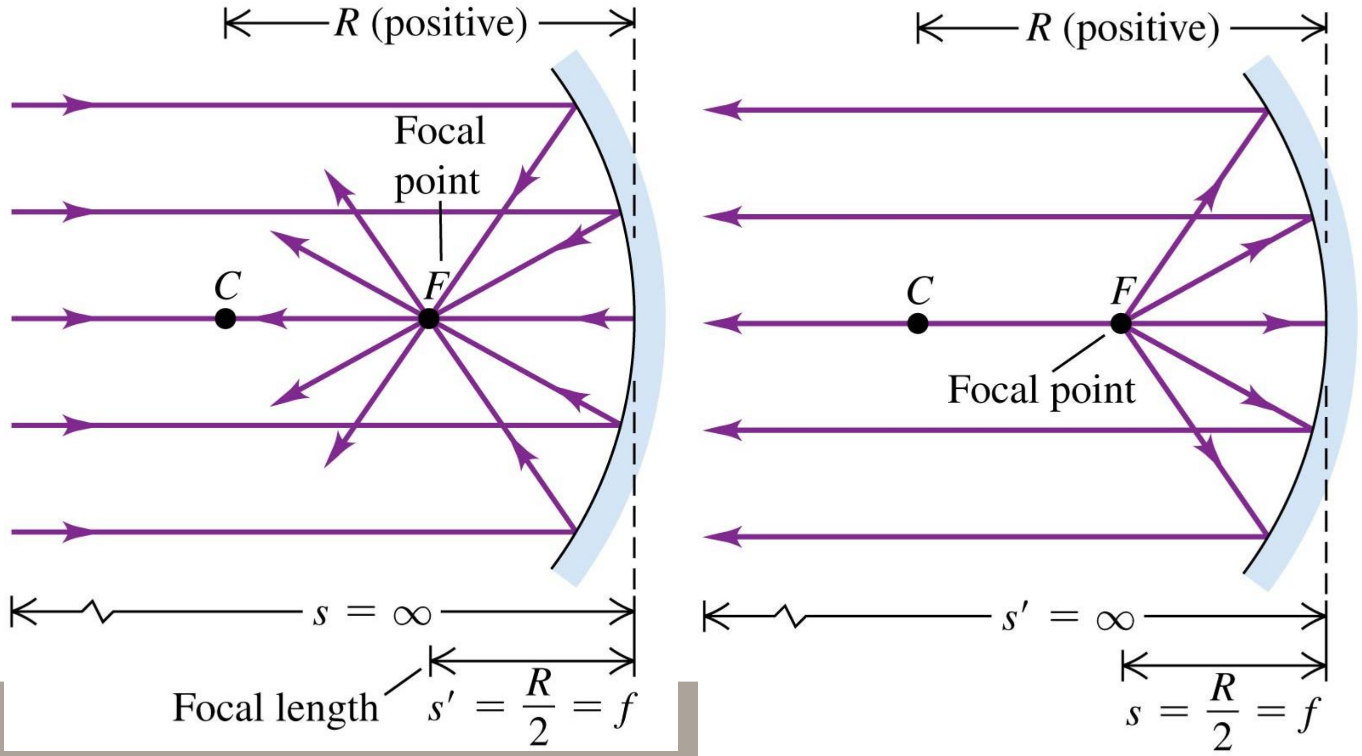
- F is called the **focal point** and f is the **focal length**.

- Rays that diverge from F will be reflected parallel.

  Mirror equation is given by:

  $$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}} .$$

- For concave mirror, s and f are **+**.



Spherical mirror, image of extended object

- The ray diagram shows us the position P', orientation, and magnification of an extended object due to reflection from a spherical concave mirror with condition $s > f$.

The magnification is:

$$\boxed{M} = \frac{y'}{y} = \boxed{-\frac{s'}{s}}$$

(image is inverted).

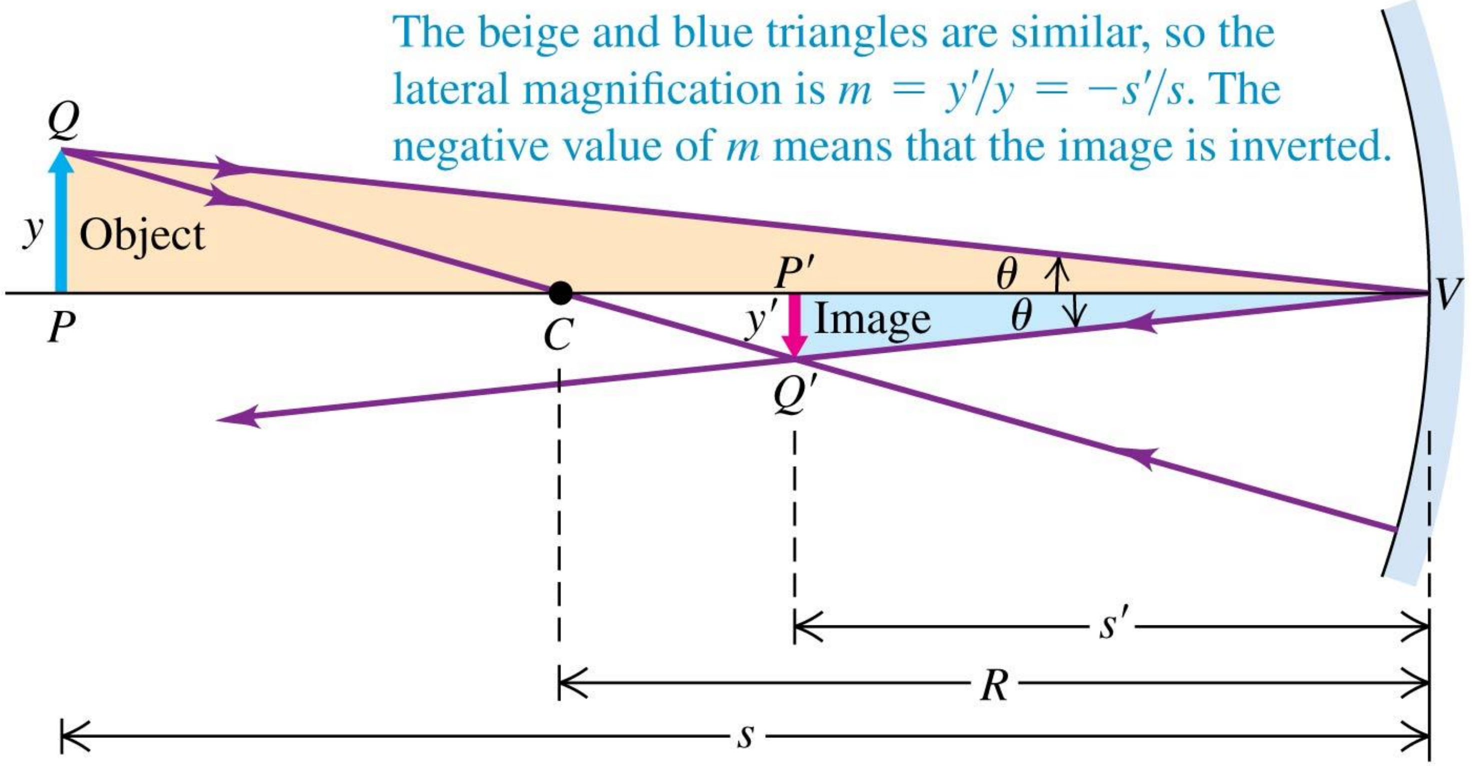
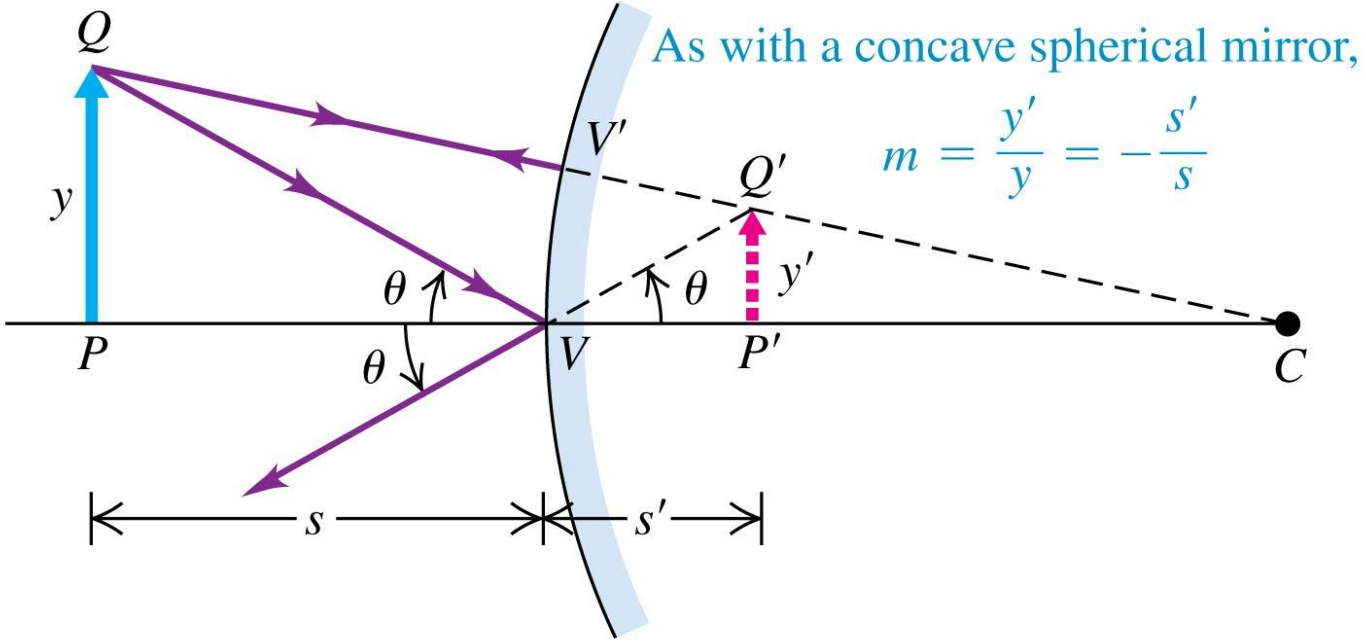


Image formation for a convex mirror

- For a **spherical *convex* mirror**, R and f are negative.

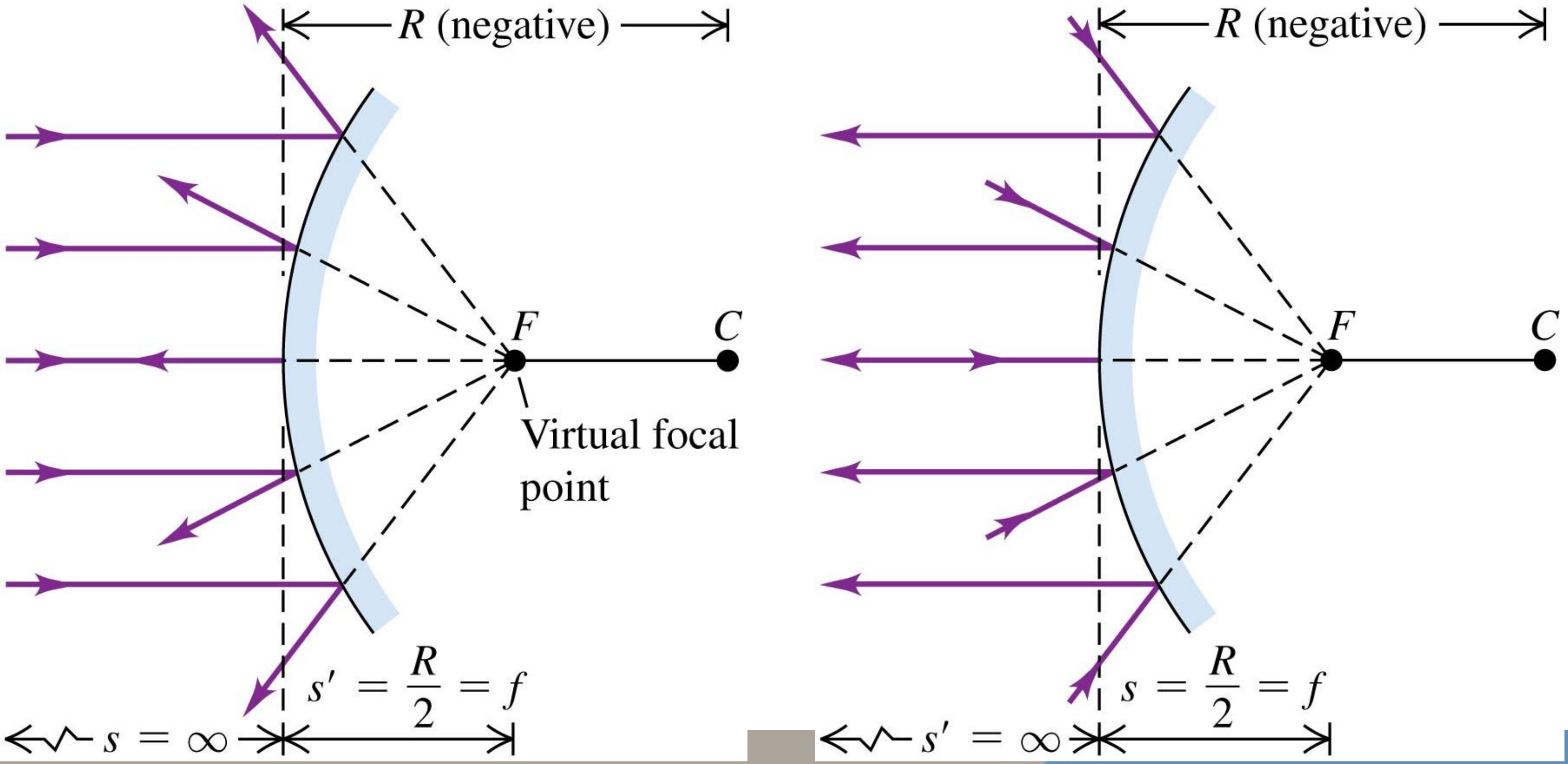
- The image distance s' is negative and the image is behind the mirror, it is a virtual image.

- The resulting image is **erect** or **upright** (M is positive).



F and f for a convex mirror

- Incoming parallel rays diverge from the virtual focal point.
- Rays pointed toward the focal point $(-F)$ are reflected parallel.



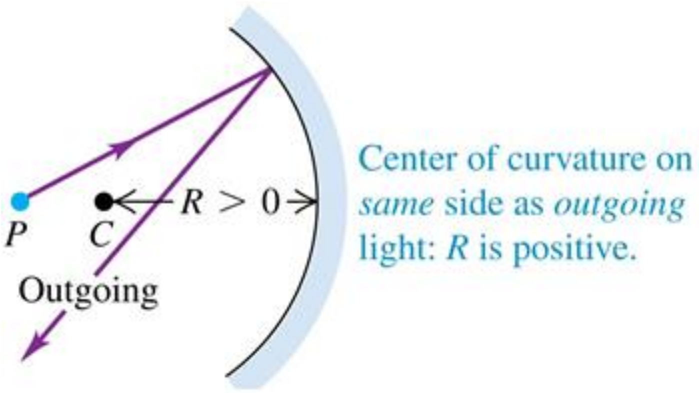
Sign convention for all mirrors

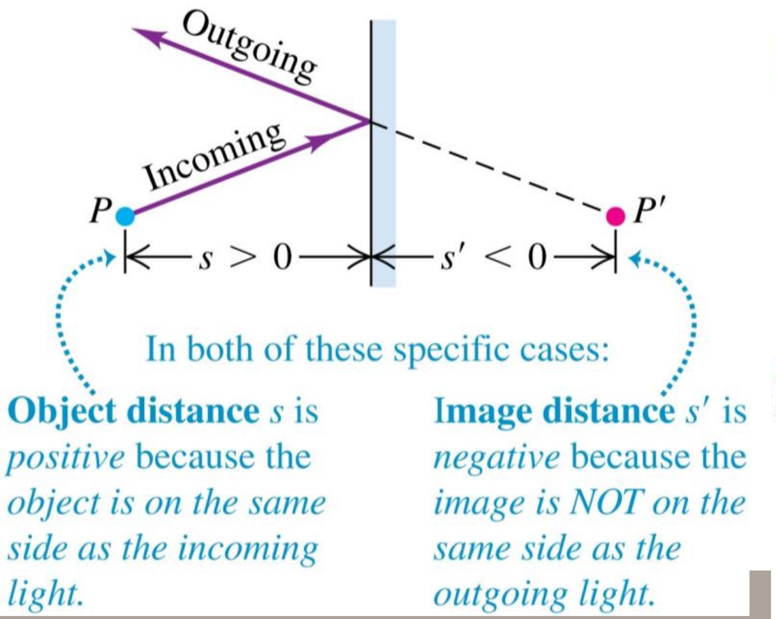
- The object (s) and image (s') distances are positive if they lie on the front side of the mirror (side where light comes in).
  - s and s' are negative if they're on the back side.

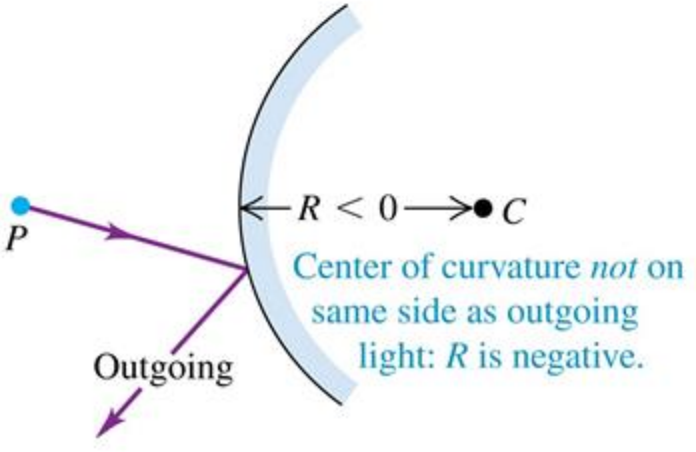
- **Concave** mirror has $+f$ and $+R$. **Convex** has $-f$ and $-R$.

- A **real** image is located on the front side of the mirror.

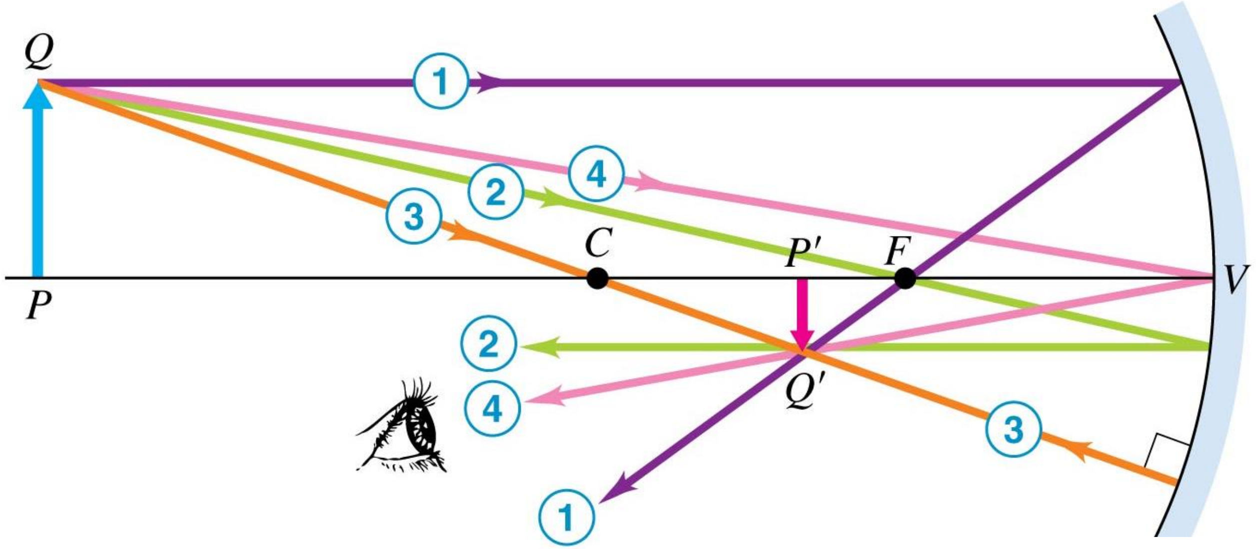
  A **virtual** image on the back side.





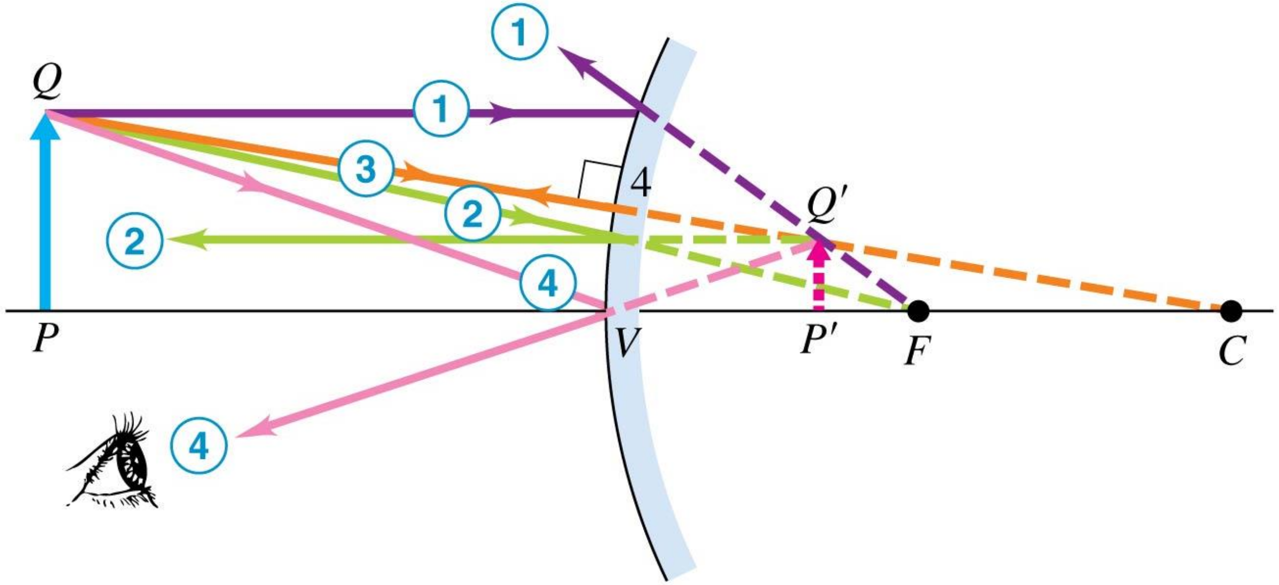


Ray diagram method of locating images



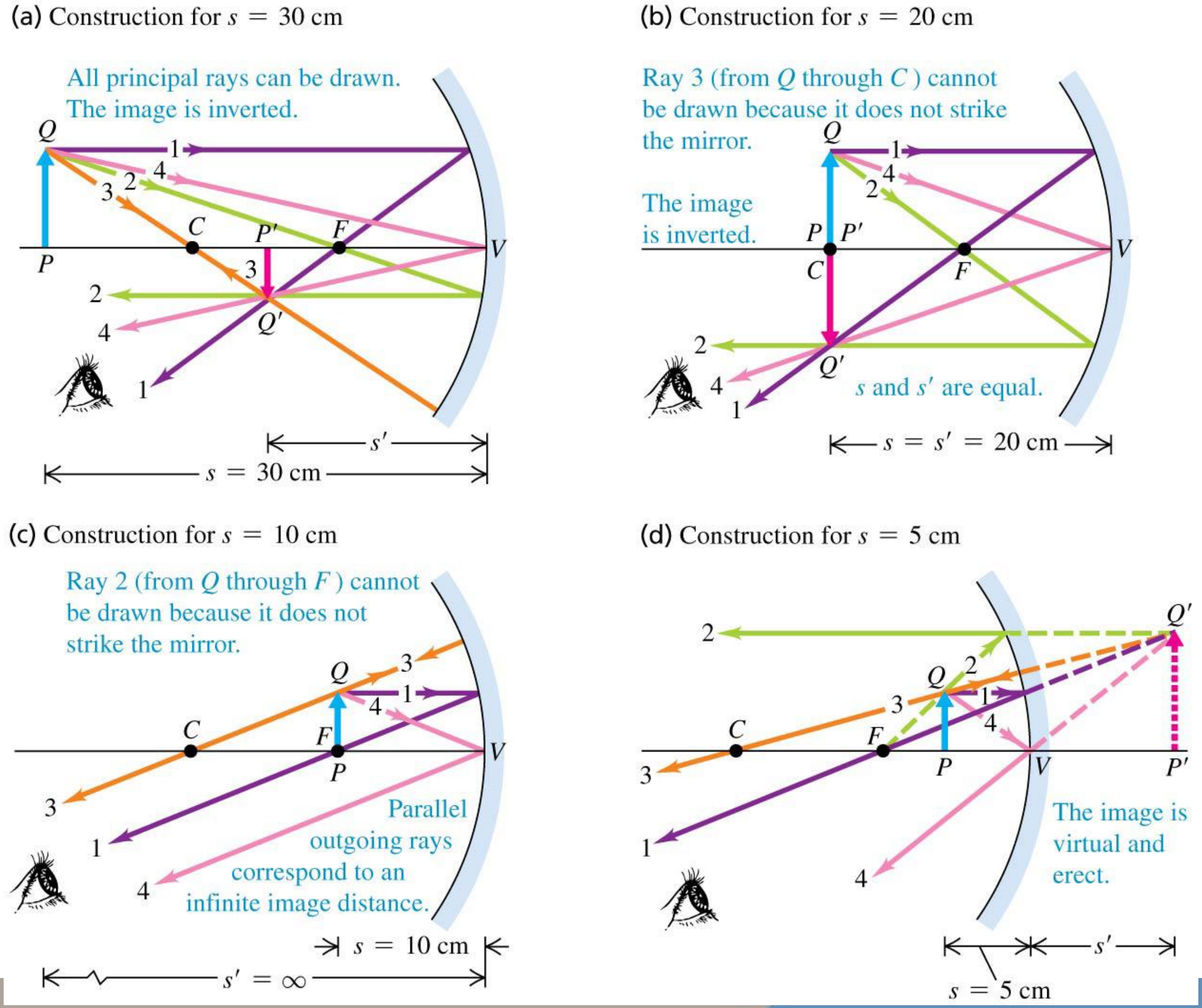
Concave

Convex



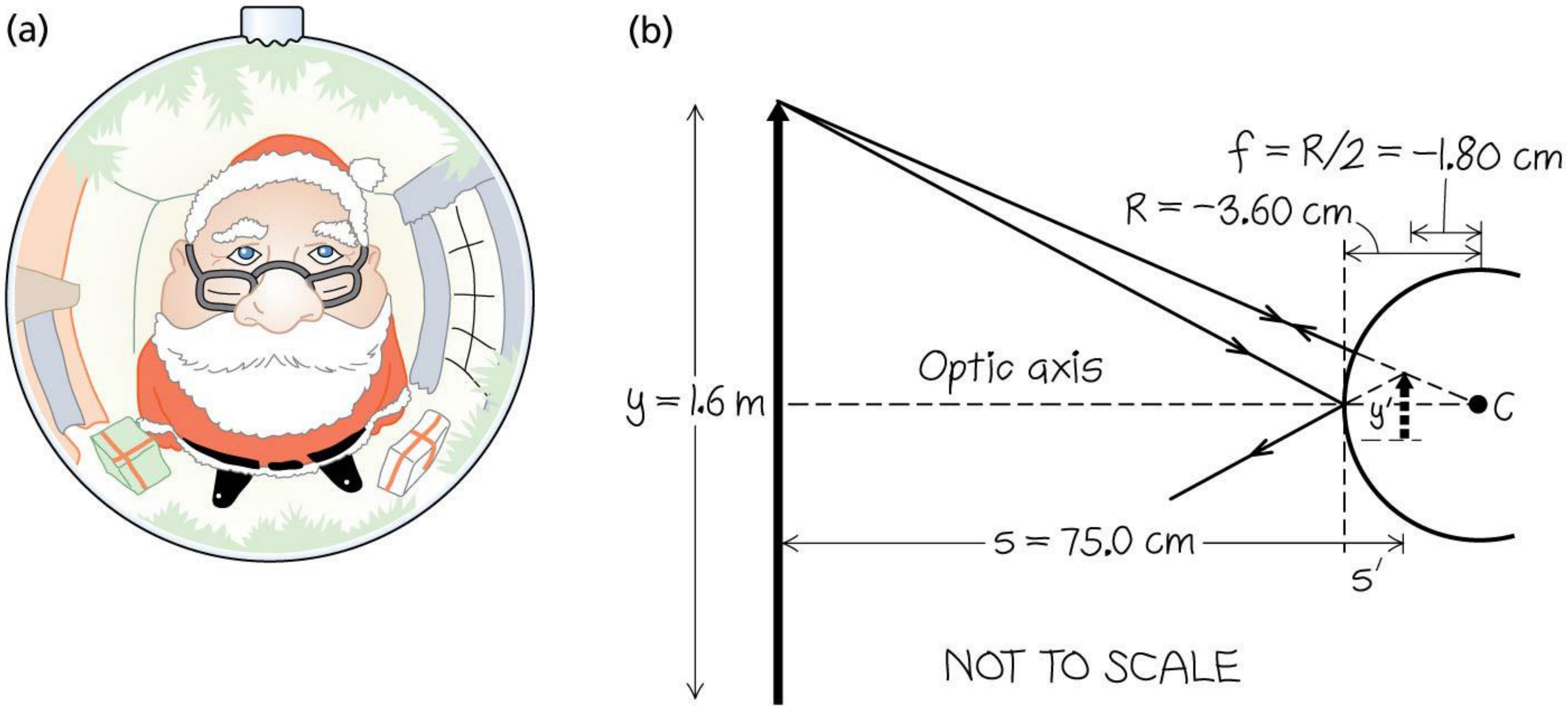
Ex. 34.4 – Concave mirror, change s

- Where does the image appear for various values of s?
- Image can be real or virtual.



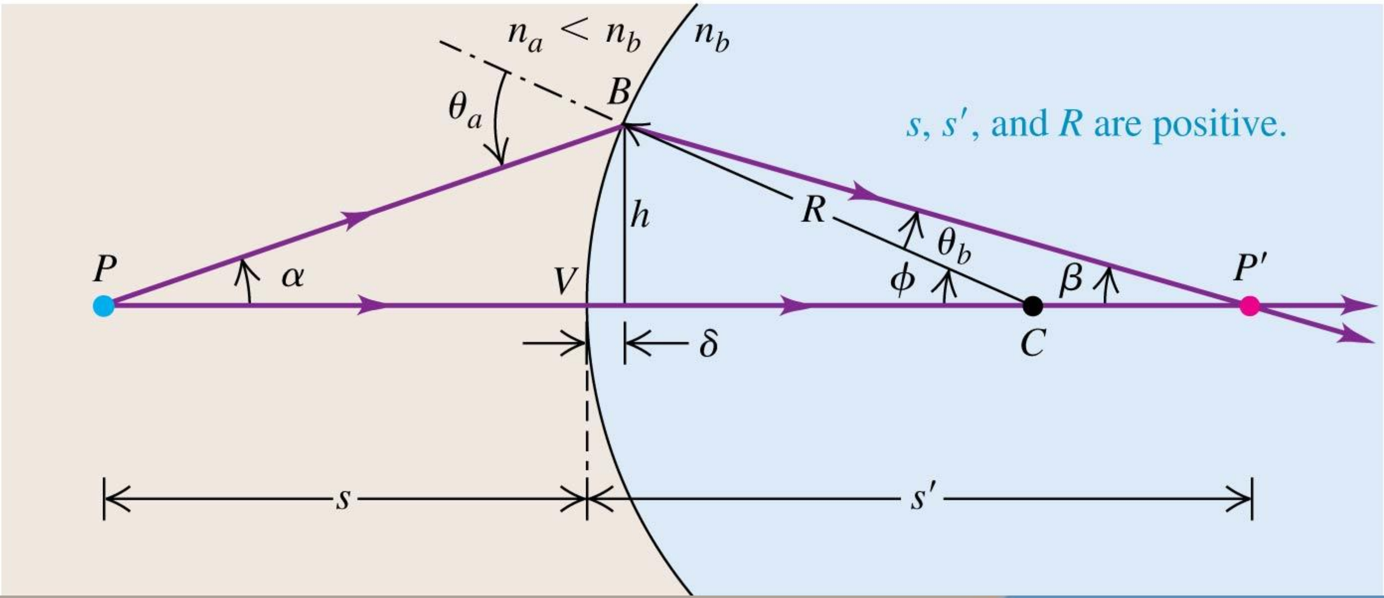
Ex. 34.3 – Santa's image problem

- Santa checks his reflection in a silvered "holiday" ornament that is 0.75 m away. The diameter of the ornament is 7.20 cm. We estimate his height to be 1.6 m tall. Where and how tall is the image of Santa formed by the ornament? Is it erect (upright) or inverted?



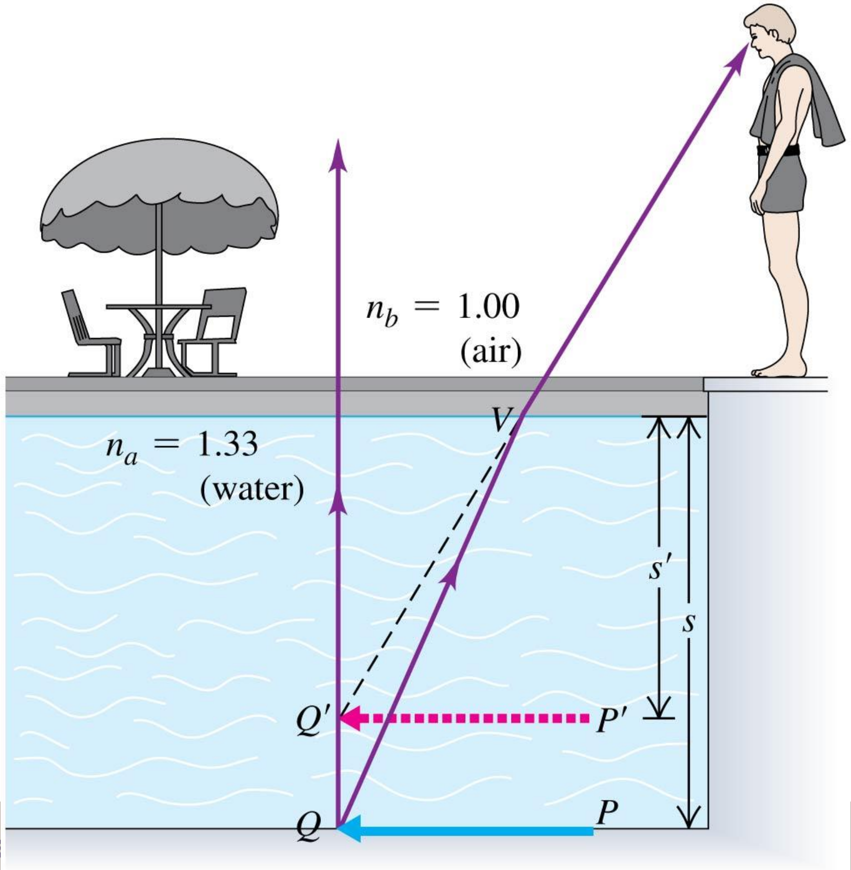
Refraction at a spherical surface

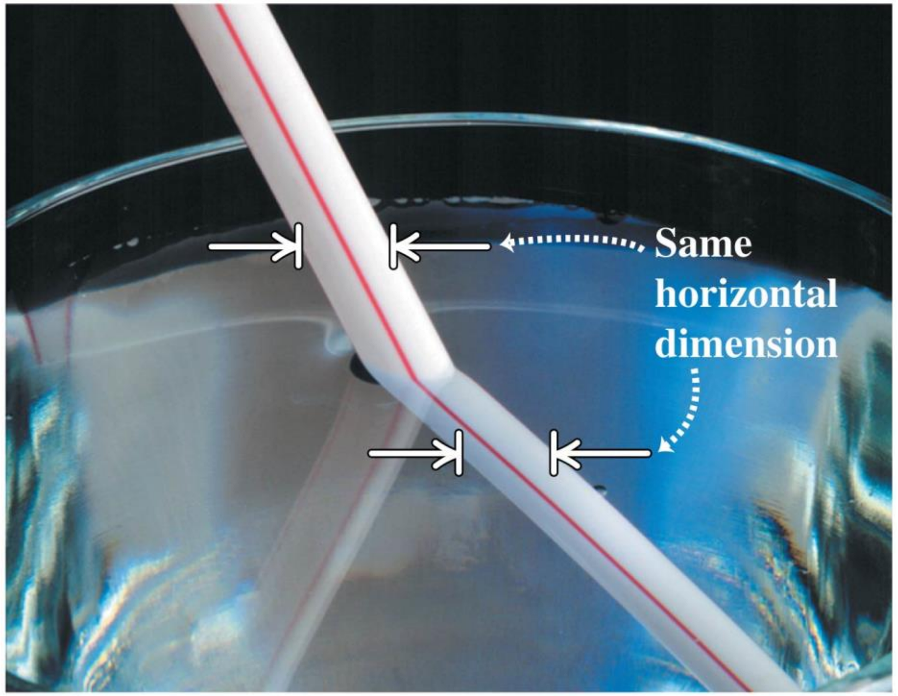
- Consider two transparent media with indices of refraction n_a and n_b. The boundary is a spherical surface with radius R.
- An image at P' inside b will be formed for an object at P in medium a.
- This principle leads to study of images formed by lenses.



Apparent depth of a swimming pool

- Image formation by refraction also leads us to believe that a swimming pool is not as deep as it is or that a straw is bent inside a glass of water.





Images formed by lenses

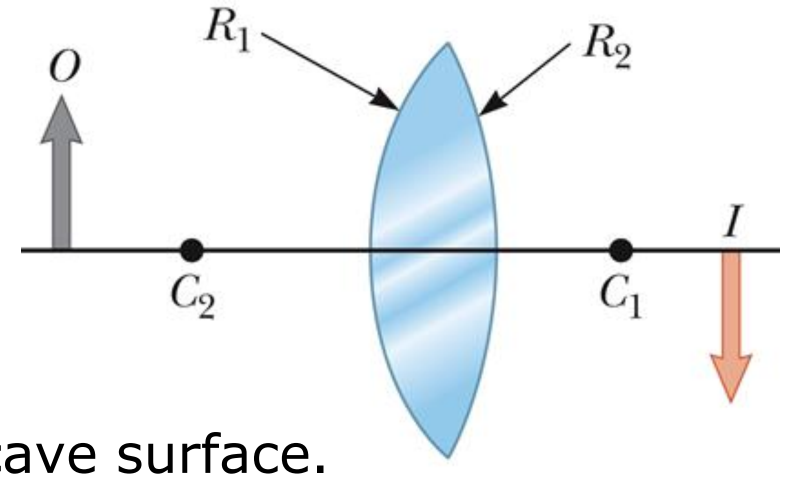
- We take what we learned from images formed by refraction and apply it to lenses.
- Lenses can be thick or thin and they are used in many optical instruments: cameras, telescopes, microscopes, binoculars…
- They occur in nature as well – ie. the lens in your eye.
- The light that passes through a lens experiences refraction at the incoming surface and the outgoing.

Geometry of a thin lens

- A thin lens will have two refractive surfaces and may have two different radii of curvature, R_1 and R_2.

- Despite having two radii, the lens has single focal length f on either side which is given by the **lensmakers' equation**.

$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$



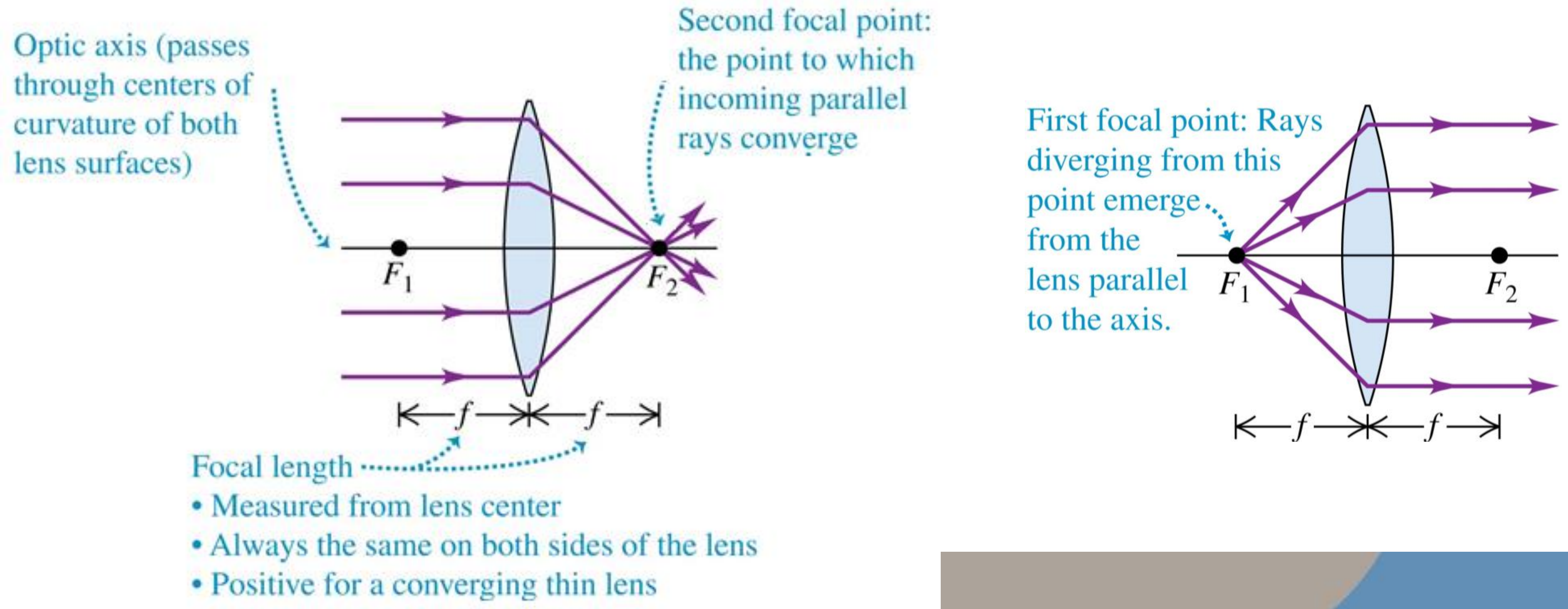
where n is the index of refraction of the lens material.

R_1 is positive for a convex surface and negative for a concave surface.
R_2 is negative for a convex surface and positive for a concave surface.

Properties of a thin (converging) lens

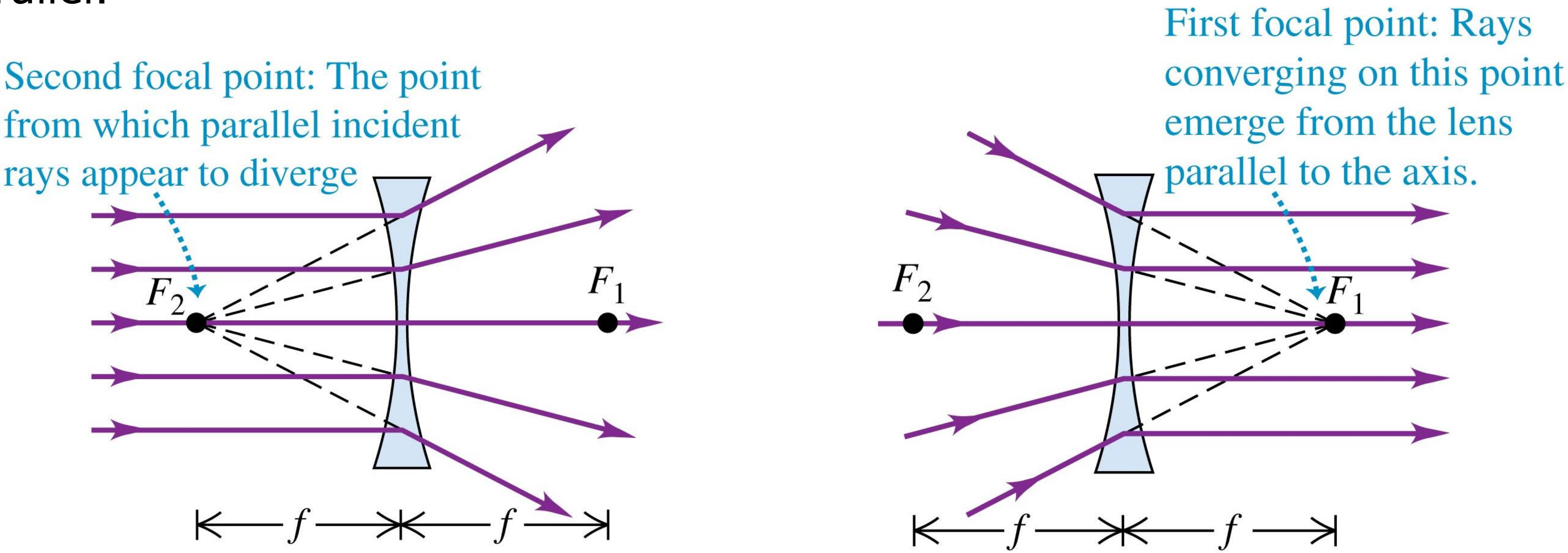
- Consider a **thin converging lens** with focal length f.

- Parallel rays will converge at a focal point F and rays emerging from F will emerge parallel, similar to a focusing (concave) spherical mirror.



Properties of a thin (diverging) lens

- A **thin diverging lens** is similar to a convex mirror.
- Parallel rays of light diverge from $+F$ and rays pointing towards $-F$ emerge parallel.



For a diverging thin lens, f is negative.

Image Formed by a thin converging lens

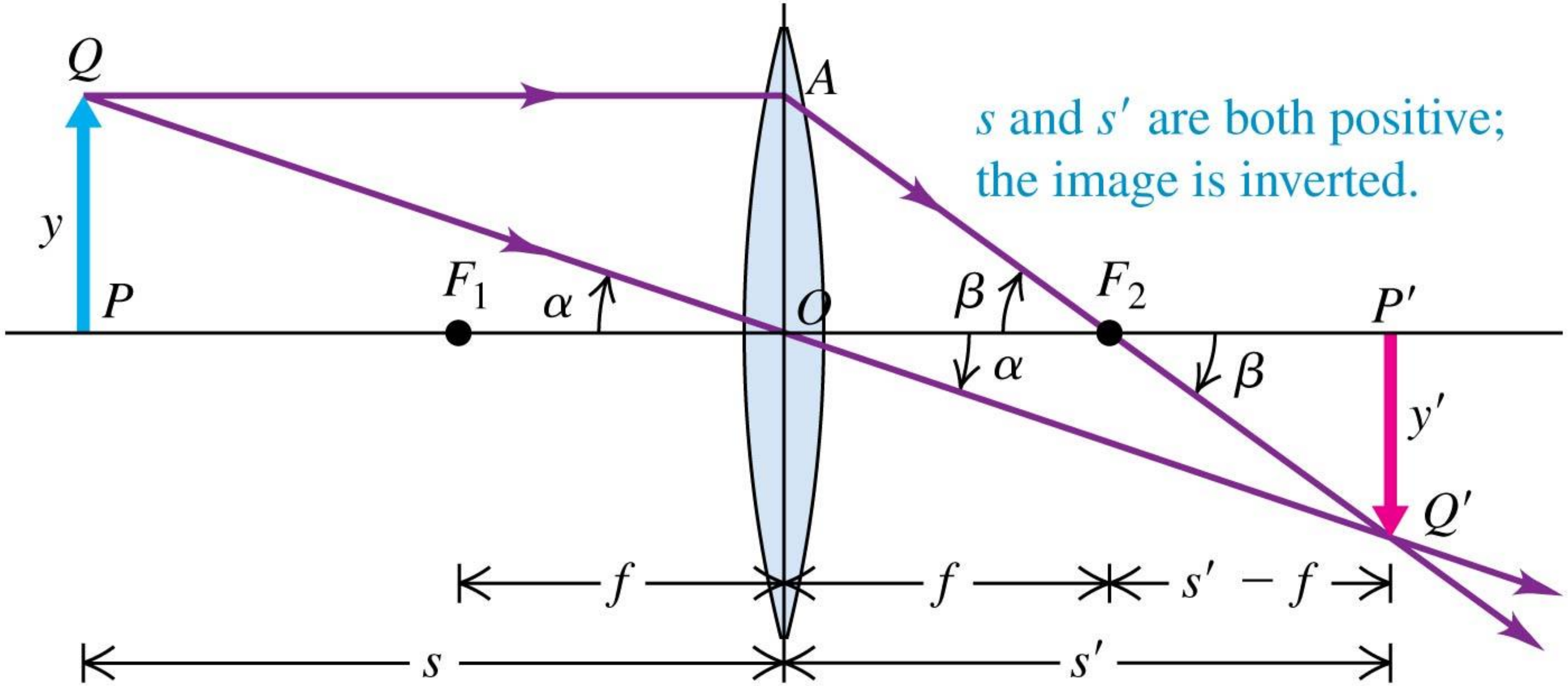
- A thin converging lens creates an image at P' for the object at P as shown in the diagram. The **thin lens equation** is:

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

which has the same form as the mirror equation.

- **The magnification is:**

$$\boxed{M} = \frac{y'}{y} = \boxed{-\frac{s'}{s}}$$



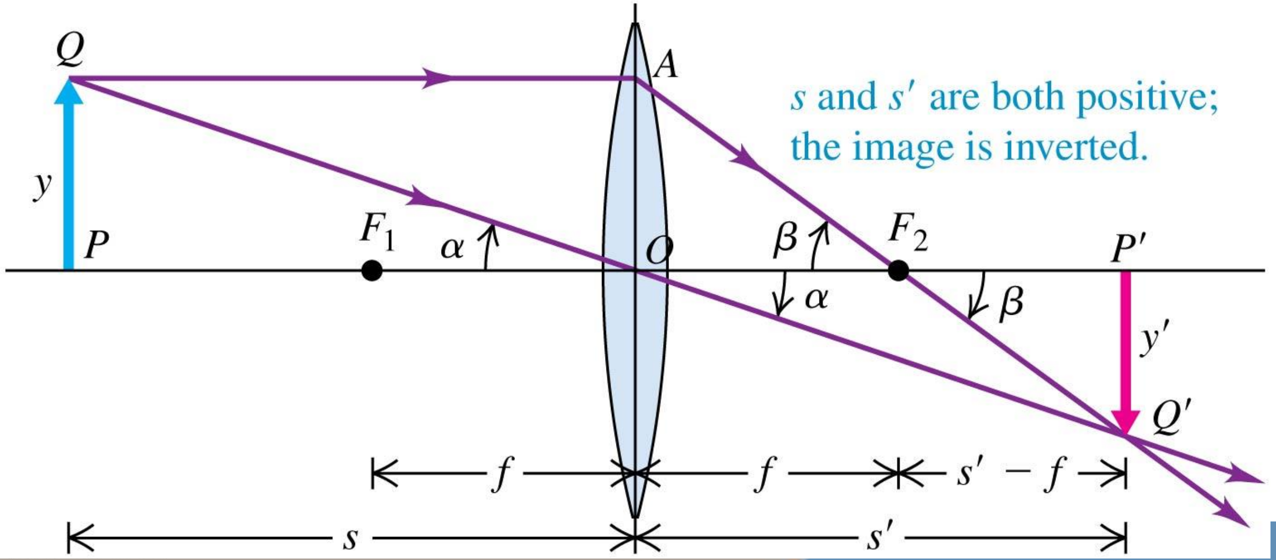
Sign convention for all lenses

- For mirrors, $+s$ and $+s'$ were located on the front side of the mirror where light came in.
  - For a lens, $+s$ refers to an object distance on the front side and $+s'$ refers to an image distance on the back side.

- Converging lenses have $+f$ while diverging lenses have $-f$.

- A *real* image is located on the back side of the lens.

  A *virtual* image is on the front side of the lens.



Types of lenses

- *Converging* lenses will focus light. They are thicker in the center than at the edges. They have a positive f.

- *Diverging* lenses cause light rays to diverge from a focal point. They are thinner at the center than at the edges and have a negative value of f.

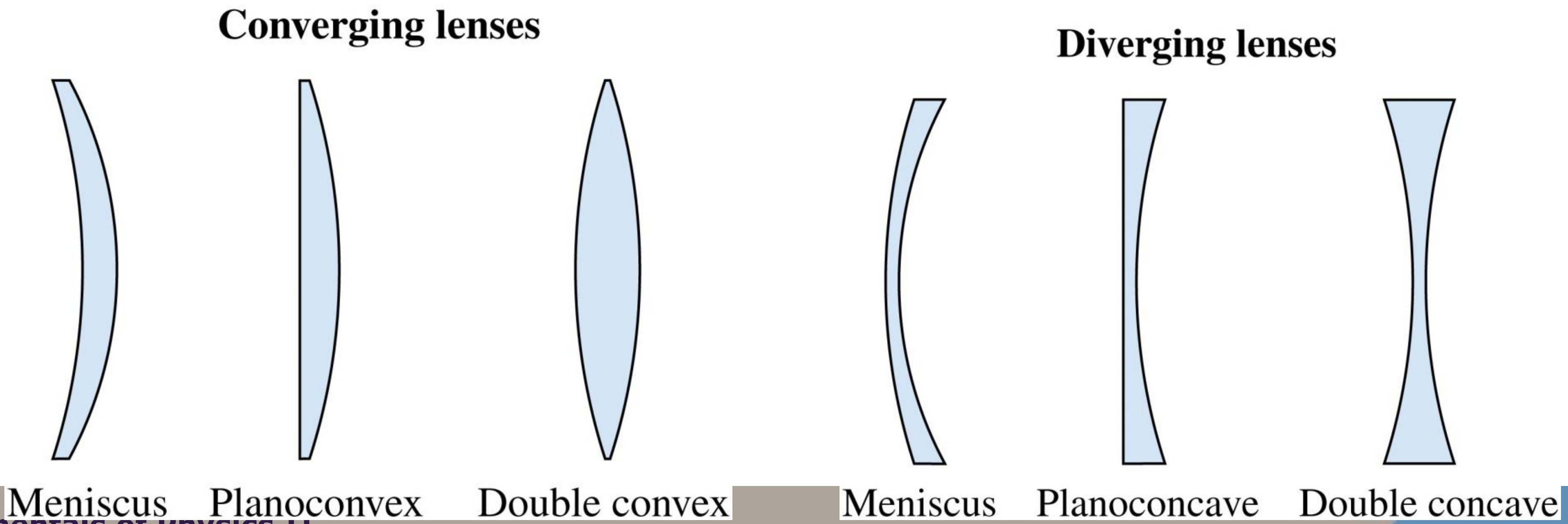
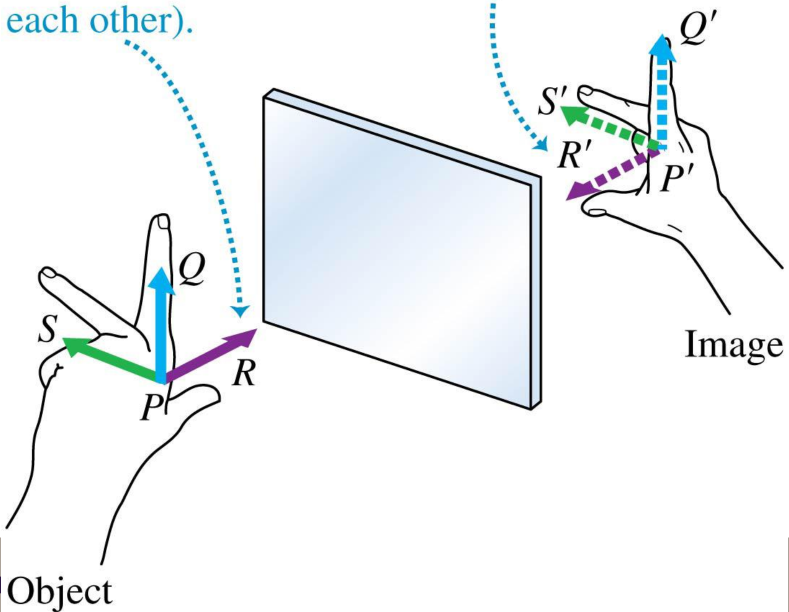


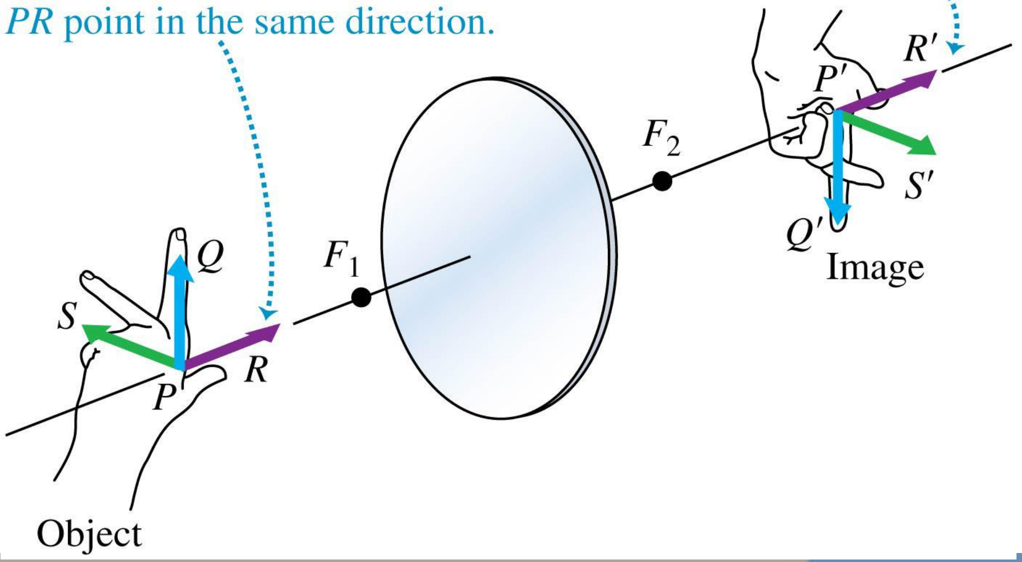
Image (non-)reversal for lenses

- While a mirror's image has the front-back reversal as previously described, the image formed by a lens does not.
- Images can still be real/virtual and upright/inverted.

An image made by a plane mirror is reversed back to front: the image thumb $P'R'$ and object thumb PR point in opposite directions (toward each other).

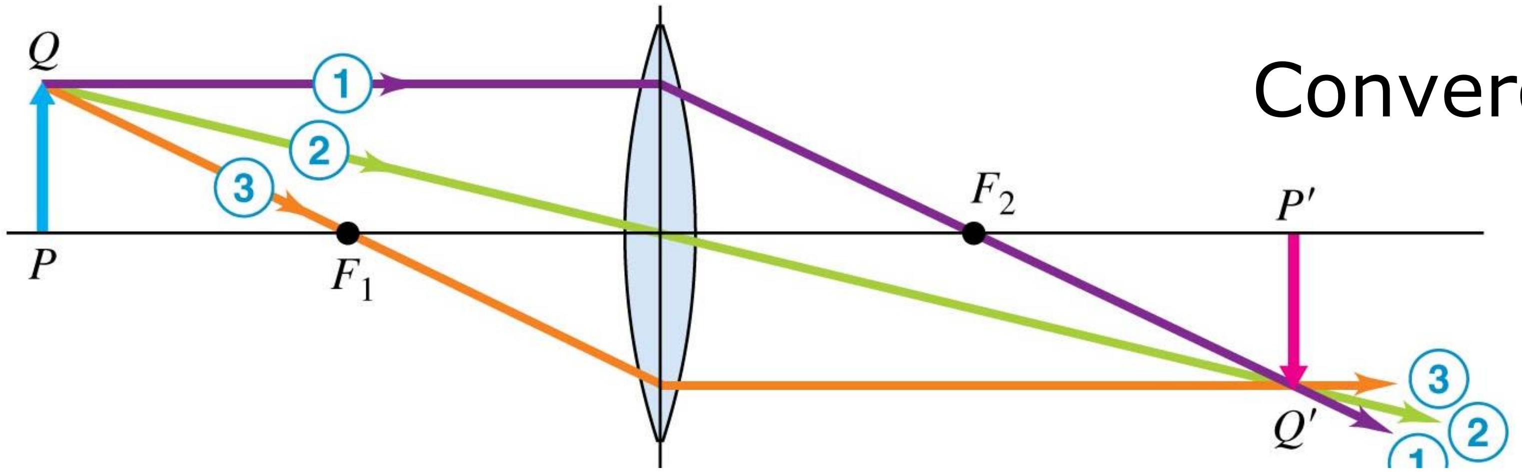


A real image made by a converging lens is inverted but *not* reversed back to front: the image thumb $P'R'$ and object thumb PR point in the same direction.

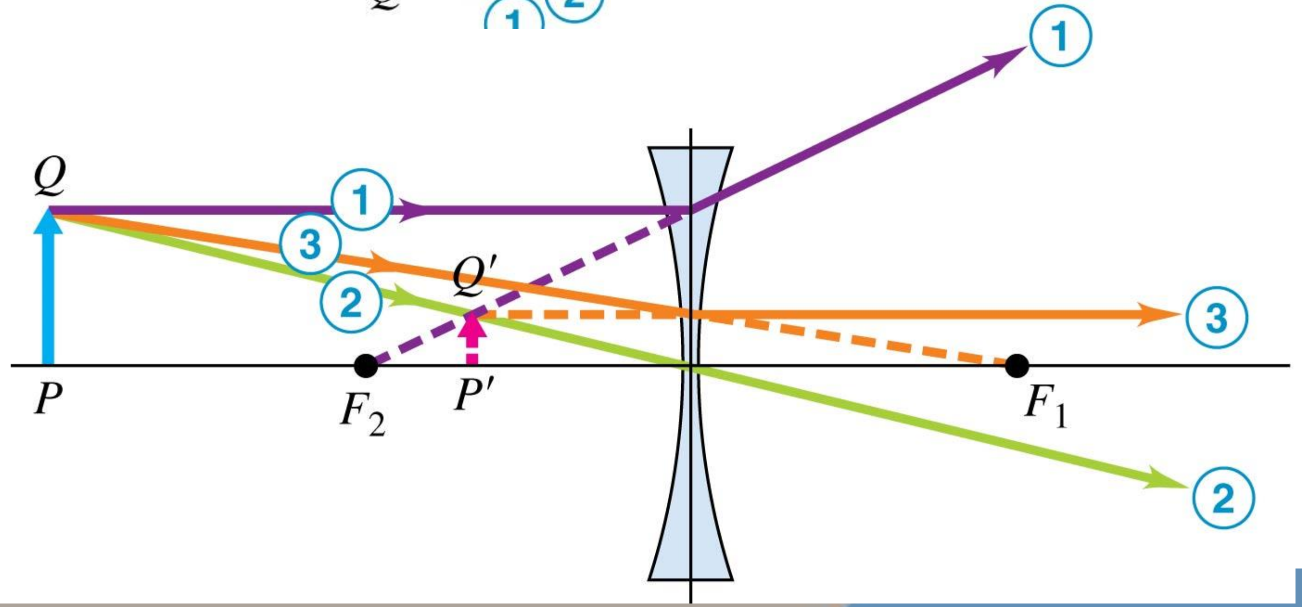


Graphical methods for lenses

Converging

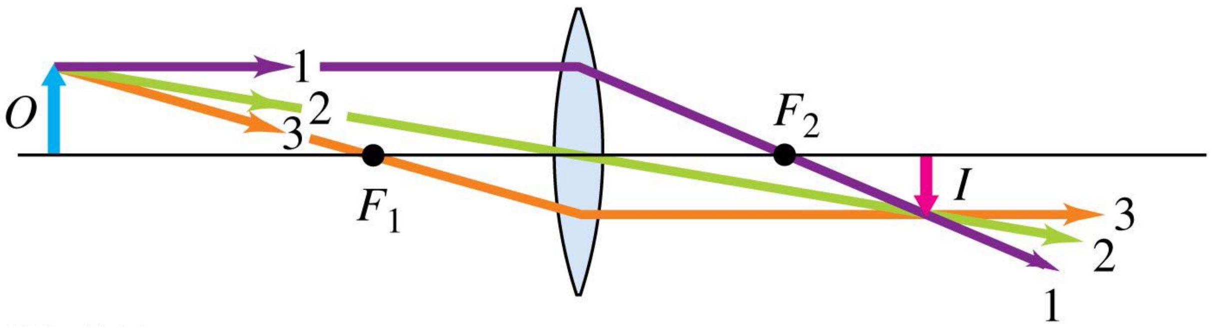


Diverging

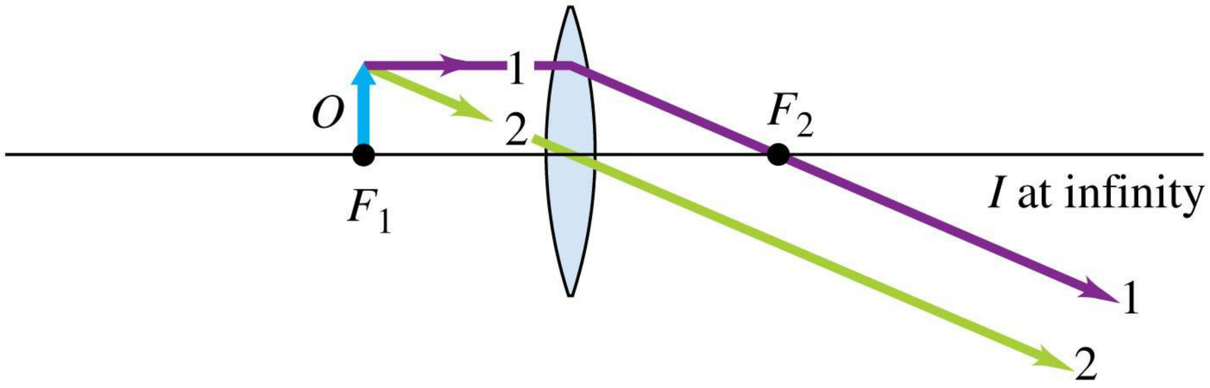


Examples of images for converging lens

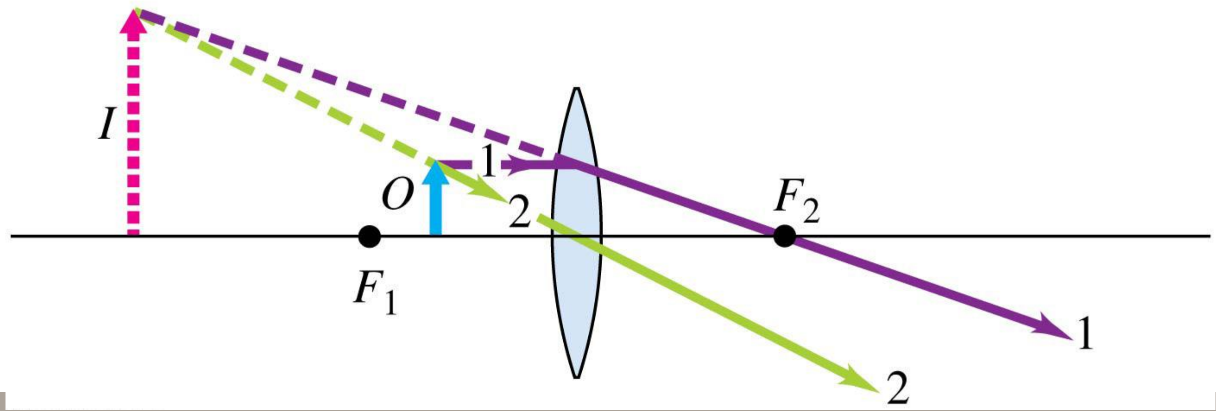
(a) Object O is outside focal point; image I is real.



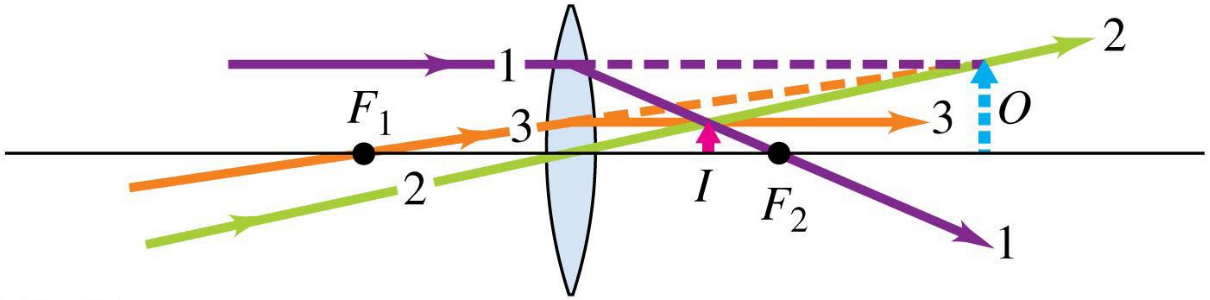
(d) Object O is at focal point; image I is at infinity.



(e) Object O is inside focal point;
image I is virtual and larger than object.



(f) A virtual object O (light rays are *converging* on lens)



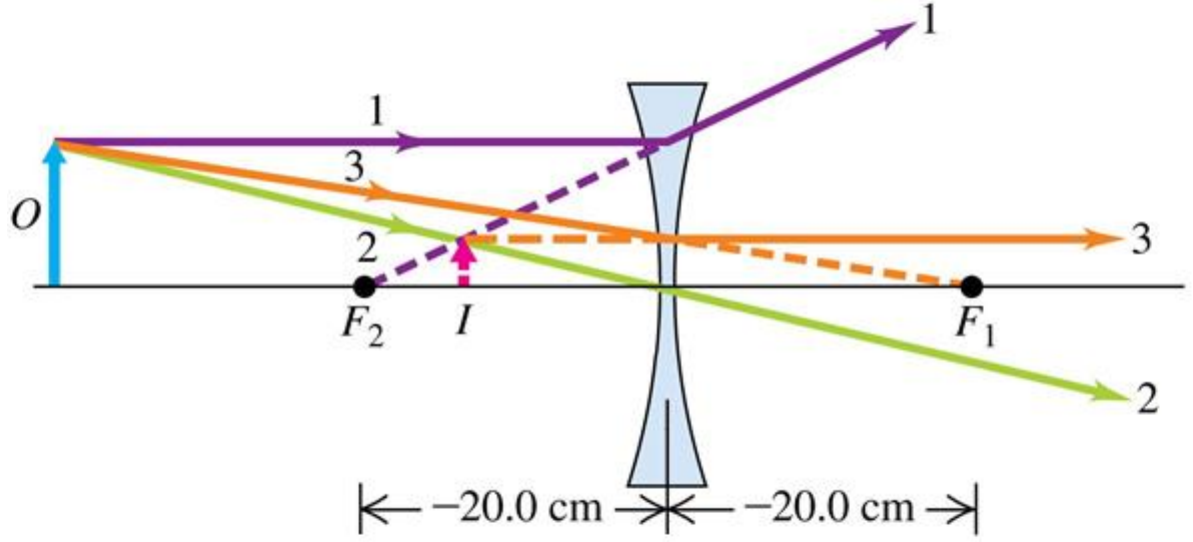
Ex. 34.10 – Image from diverging lens

- A beam of parallel rays spreads out after passing through a thin diverging lens as if they all came from a point that is 20.0 cm from the center of the lens.

  You want to use this lens to form an upright, **virtual** image that is 1/3rd the height of the object.

  (a) Where should the object be placed? Where will the image be?

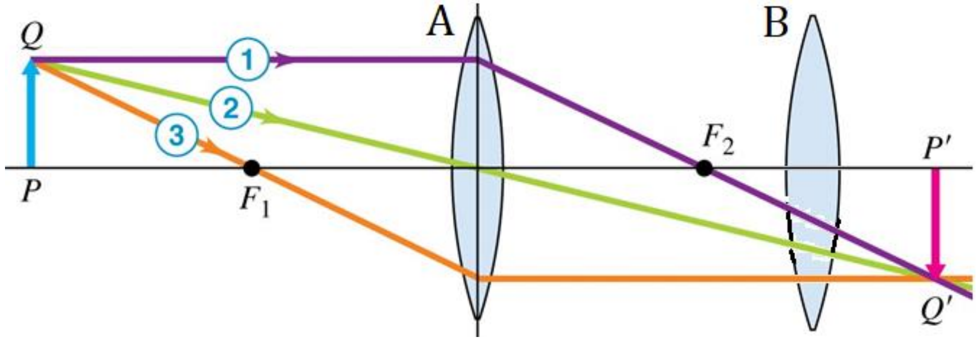
  (b) Sketch a principal ray diagram (shown to the right).



Combination of thin lenses

- If two thin lenses are used to form an image, the system is treated like so:
  1) Find image formed by the first lens (s'_A)
  2) Use image of first lens as the object for the second lens.
  3) Find the image for second lens (s'_B). You're done!

- If the image of the first lens is on the back side of the second lens then it is treated like a *virtual object* (s_B is **negative**).



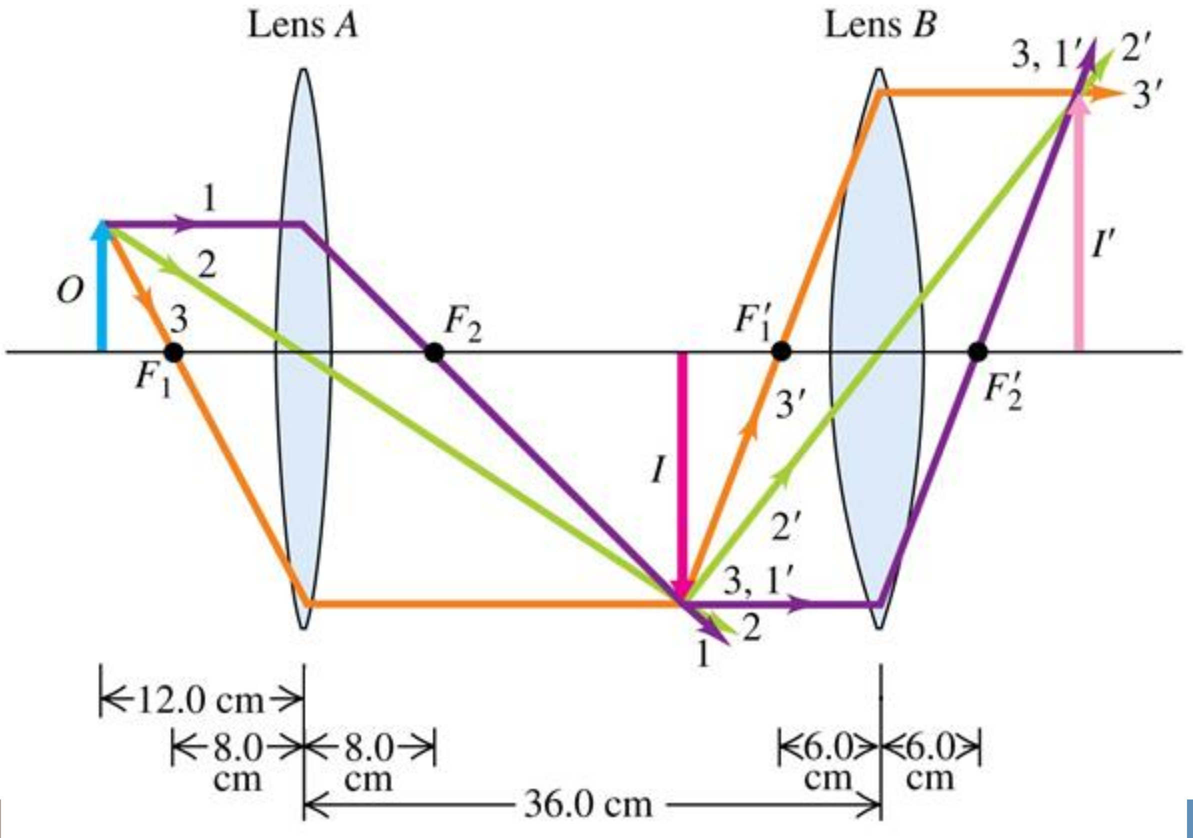
- The overall magnification will be the product of each individual magnification:

$$\boxed{M} = M_1 M_2 = \boxed{\left(-\frac{s'_A}{s_A}\right)\left(-\frac{s'_B}{s_B}\right)}$$

Ex. 34.11 – An image of an image

- Converging lenses A ($f_A = 8\ \mathrm{cm}$) and B ($f_B = 6\ \mathrm{cm}$) are placed 36.0 cm apart. Both lenses have the same optic axis. An object 8.0 cm high is placed 12.0 cm to the left of lens A.

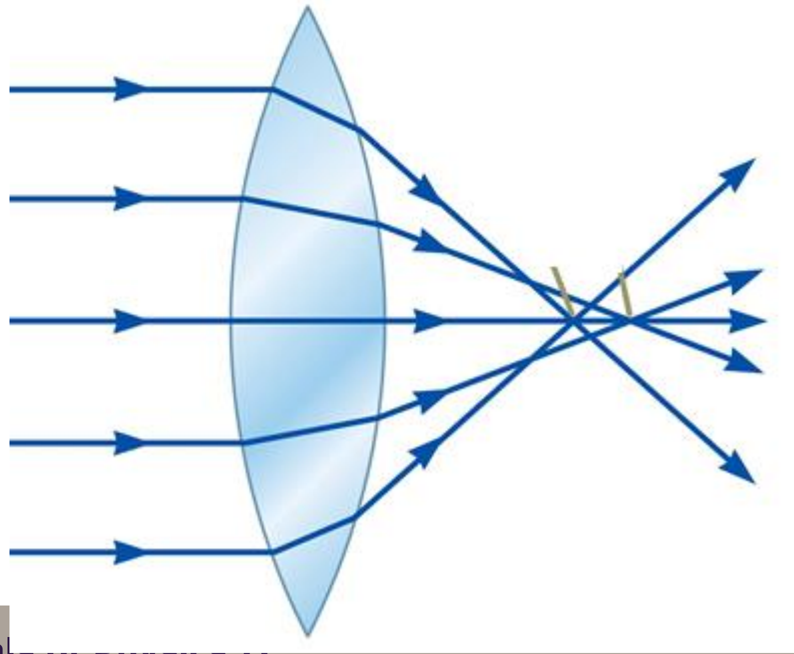
  Find the position, size, and orientation of the image produced by the lenses in combination.

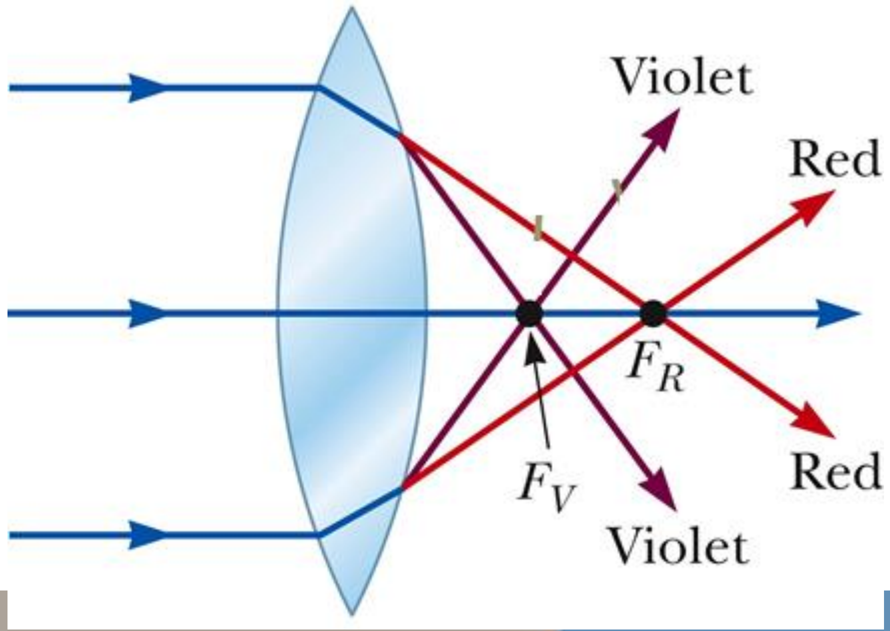


Aberrations (spherical and chromatic)

- Aberrations due to lens shape (spherical) or refractive index dependence (chromatic) can cause blurriness in images.

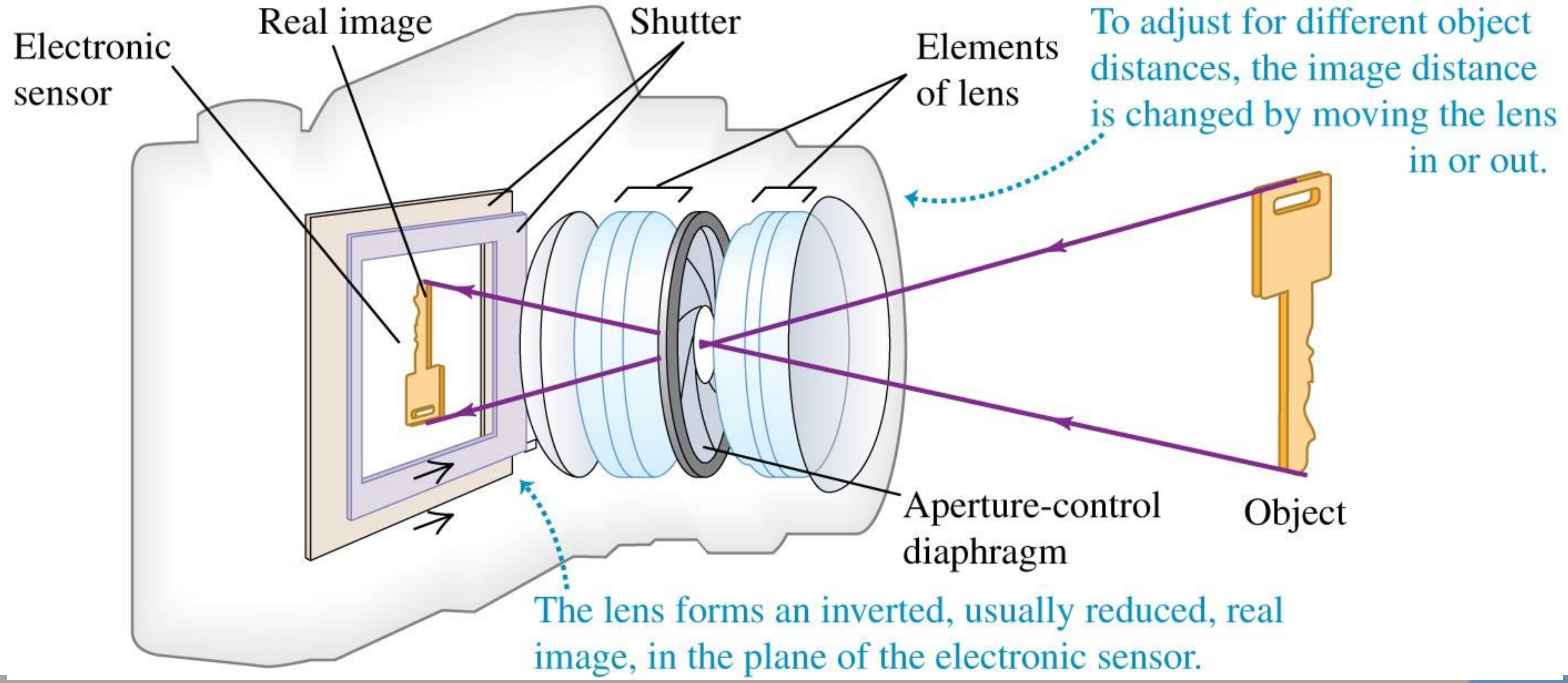
- This can be corrected by using parabolic lenses or adding extra material to the back side of a lens (to create a doublet).





Cameras

- When a camera is in proper focus, the position of the electronic sensor coincides with the position of the real image formed by the lens setup.



Using different camera lenses

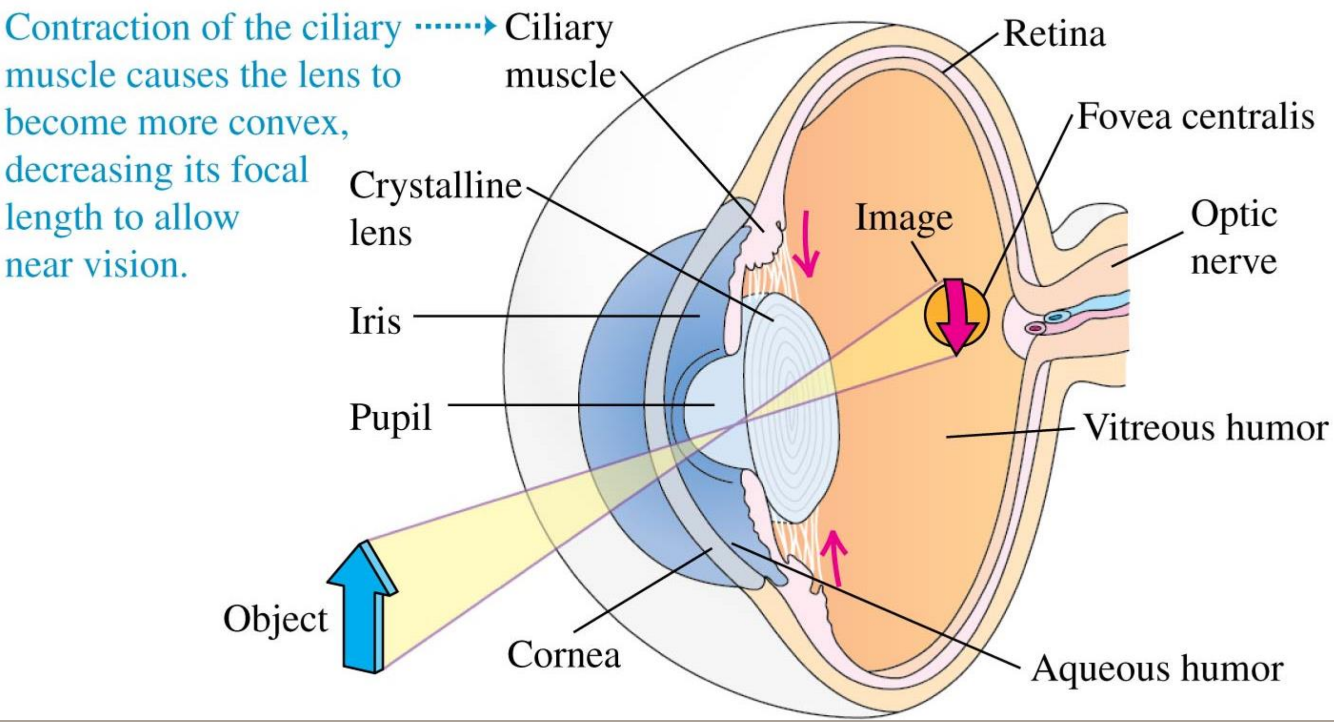
- Using lenses with different focal lengths allows you to change the magnification of the image you are capturing without changing the distance you are from the object.

(a) $f = 28$ mm (b) $f = 70$ mm (c) $f = 135$ mm

- A camera lens makes an inverted image on the camera's sensor. A digital camera's internal software re-inverts the image so you can easily view your picture on a display.

The eye

- The optical behaviour of the human eye is like that of a camera. Your eye has a lens at the front which focuses light onto the retina. The regions in front of and behind the lens contain liquid. The setup ideally allows humans to have clear and sharp vision.



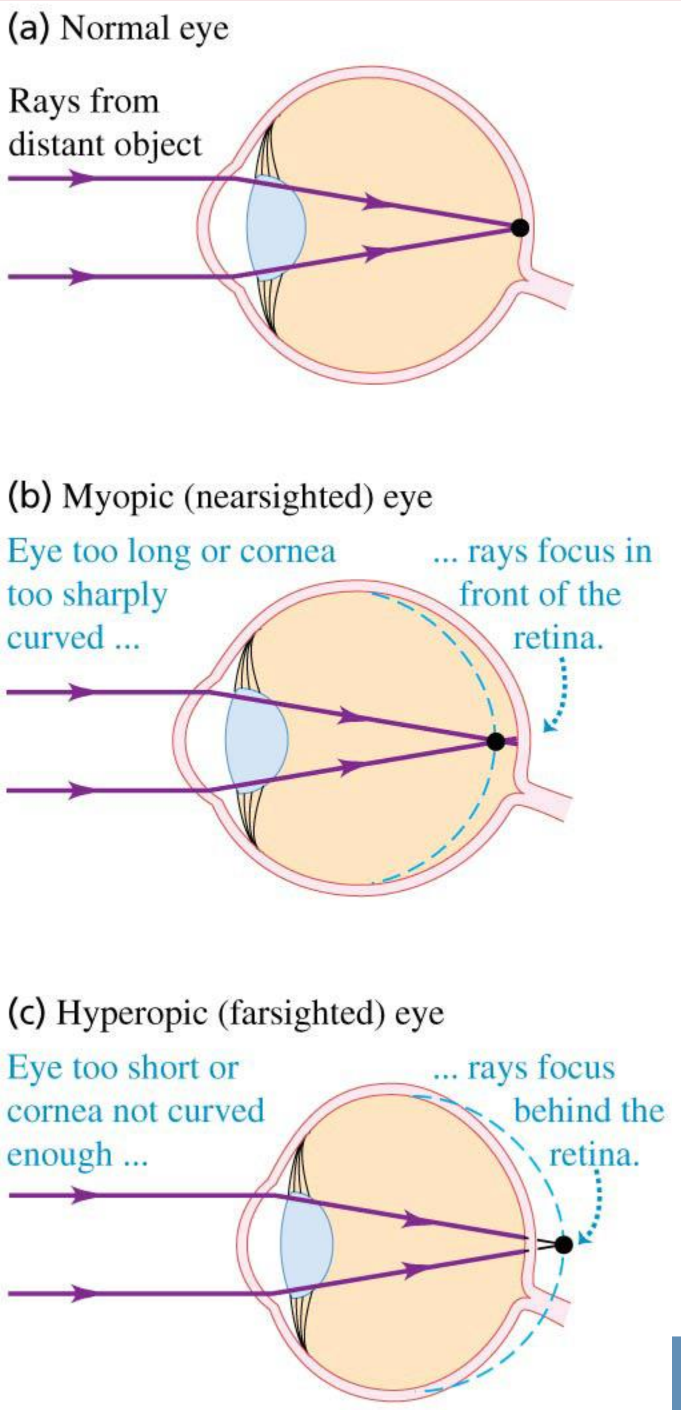
Defects in vision

- Different defects in vision can occur such as:

  Myopia (nearsightedness) where the eyeball is too long and the lens focuses light in front of the retina.

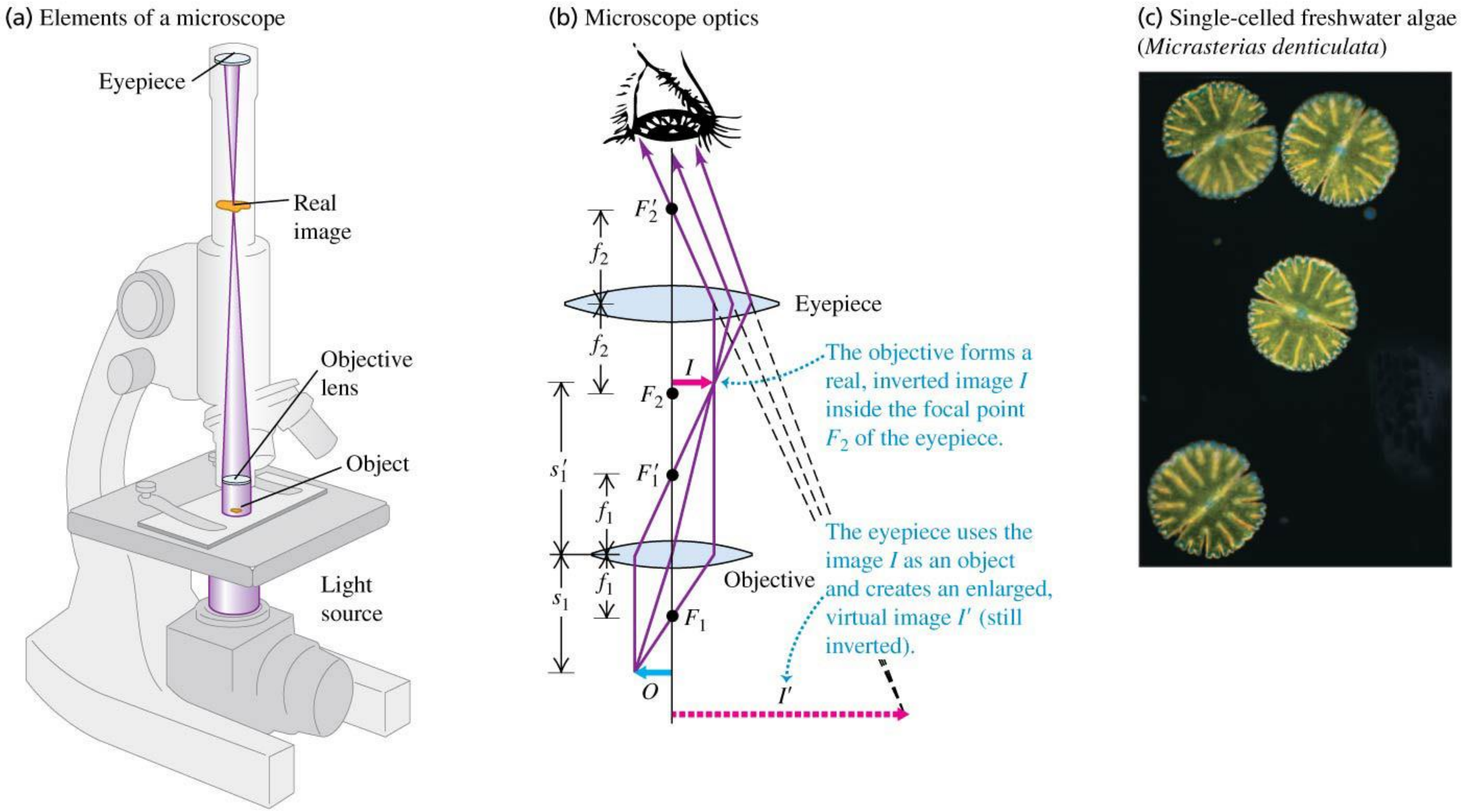
  Hyperopea (farsightness) where the eye is too short or there's not enough curve in the cornea.

- We can correct these defects by wearing lenses (glasses) or with surgery (shaping the cornea with a laser).



Microscope

- A microscope is used to make a large magnification of a small object.



Telescope

- A telescope allows one to image a far away object using a simple system of lenses or mirrors.