Chapter 21

- Electric charge and electric field

Michael Wong – PHY 1122 Spring 2023

Learning goals

- Electric charge, conservation of charge
- Coulomb's law and applications
- Relationship between electric force and field
- Using electric field lines
- Properties of electric charge distributions

Electricity and magnetism

- Originally thought to be two separate forces - eventually unified by Maxwell (1873).
- Branch of physics involving the study of electromagnetic force.
- History of EM in text, read in textbook (or Wikipedia) if you're interested!
- The **electromagnetic force** is a field force between objects with charge (like gravity for things with mass).

Electric charge

- Electric charge is an "invisible" fundamental property of matter that causes it to experience a force in an electromagnetic field.
- Electric charge (q) is **quantized** (like money, unlike time) and an object can carry units (C) of charge.
- Objects can have **negative** or **positive** charge or be **neutral**.
- **Like** charges **repel**, <u>opposite</u> charges <u>attract</u>.
- Electric charge is **conserved**.

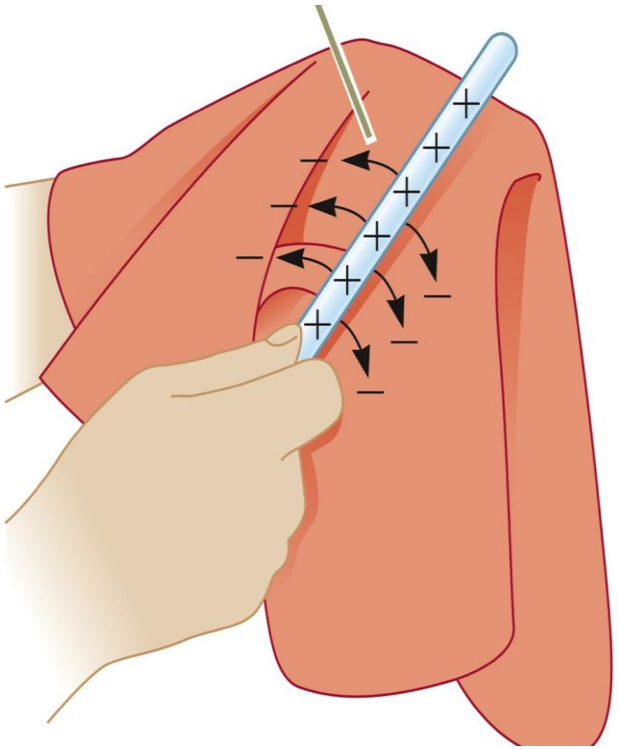
Conservation of electric charges

In an isolated system:

- Electric charge is conserved, not created!

- Rubbing of silk and glass:
  - Silk becomes negatively charged.
  - Glass becomes positively charged.

- Only electrons move!

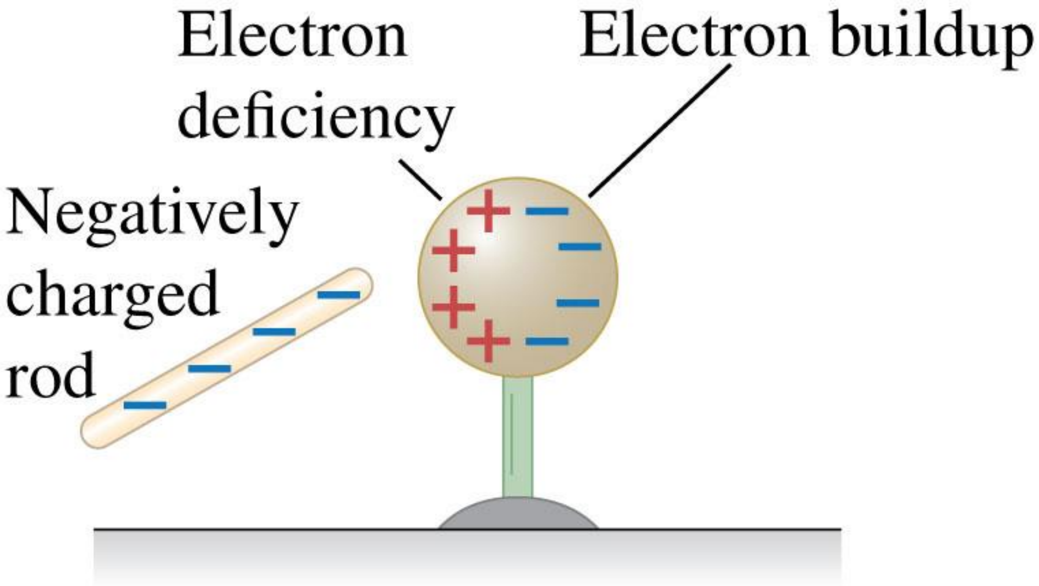


Induced charges

Three types of materials:

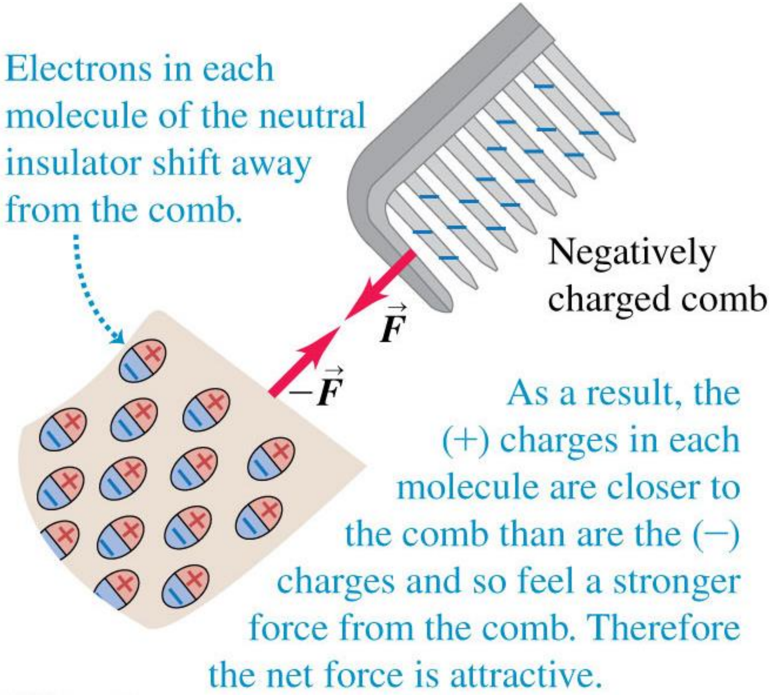
- **Conductors**, **insulators**, and semiconductors.

- Charging by **induction** is different than conduction.



Electric forces on uncharged objects

- Induced charge effect and **polarization** in the uncharged object.
- Examples: balloon against the wall, comb and paper.



Coulomb's Law

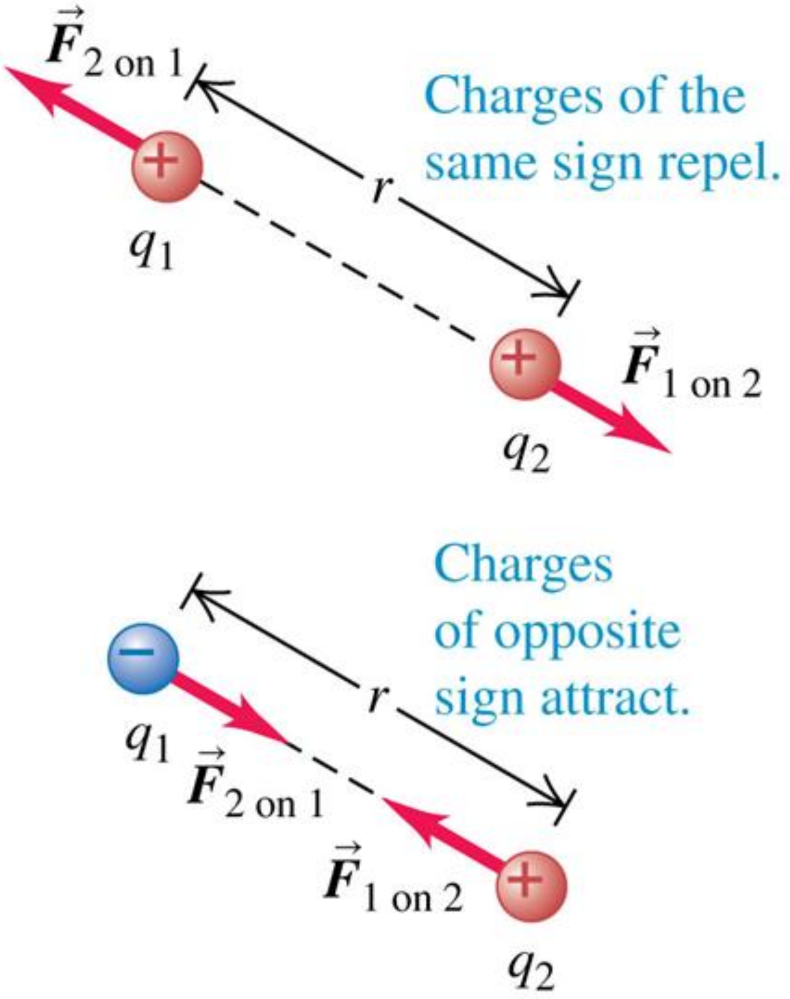
- Coulomb's Law for two point charges:

$$\boxed{F = k\frac{|q_1||q_2|}{r^2}} \text{ (in N)}$$

where $k = 8.9876 \times 10^9 \, \frac{\mathrm{Nm}^2}{\mathrm{C}^2}$

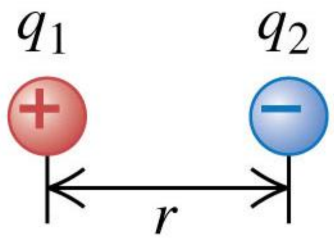
$$\boxed{k = \frac{1}{4\pi\epsilon_0}}$$

  - SI unit of charge is the coulomb (1 C)
  - $e = 1.602 \times 10^{-19}$ C



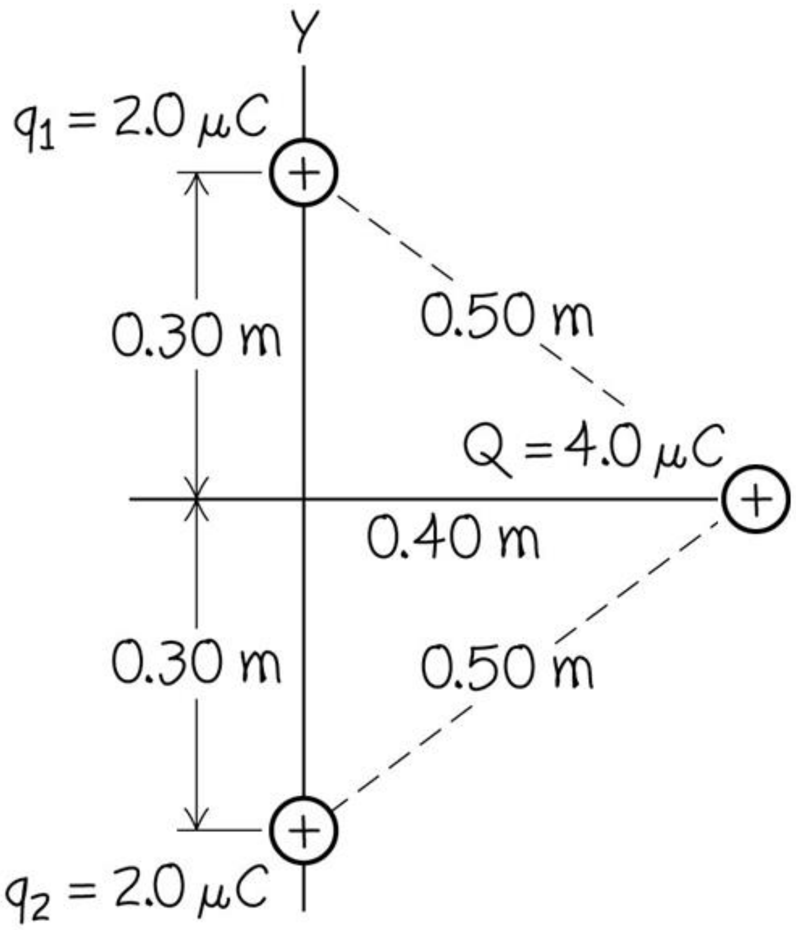
Ex. 21.2 - Force between 2 point charges

- Two point charges, $q_1 = +25\ \mathrm{nC}$ and $q_2 = -75\ \mathrm{nC}$, are separated by a distance $r = 3.0\ \mathrm{cm}$. Find the magnitude and direction of the electric force
  (a) that q_1 exerts on q_2 and
  (b) that q_2 exerts on q_1



Ex. 21.4 – Addition of electric forces

- Two equal positive charges $q_1 = q_2 = 2.0\ \mu\mathrm{C}$ are located at $(x, y) = (0, 0.30)\ \mathrm{m}$ and $(x, y) = (0, -0.30)\ \mathrm{m}$, respectively. What are the magnitude and direction of the total electric force that q_1 and q_2 exert on a third charge $Q = 4.0\ \mu\mathrm{C}$ at $(x, y) = (0.40, 0)\ \mathrm{m}$?



Exercise 21.6 - # electrons

- Two small spheres spaced 20.0 cm apart have equal charge. How many excess electrons must be present on each sphere if the magnitude of the force of repulsion between them is 3.33×10^{-21} N?

- NB. $e = -1.6 \times 10^{-19}$ C is the charge of one electron

Electric field

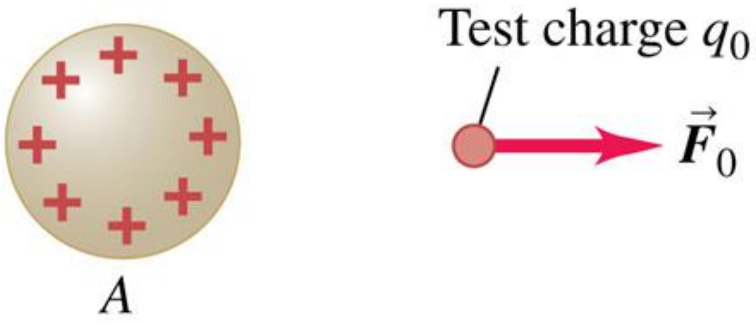
- Consider the positively charged A with charge Q and the point P.
  - If you put a test particle with charge q_0 at P, it feels force $\vec{F}_0$.

- Because object A has charge, it generates an **electric field** around it. At P, the magnitude is:

$$\boxed{E = k\frac{Q}{r^2}} \text{ with units } \frac{\mathrm{N}}{\mathrm{C}}$$

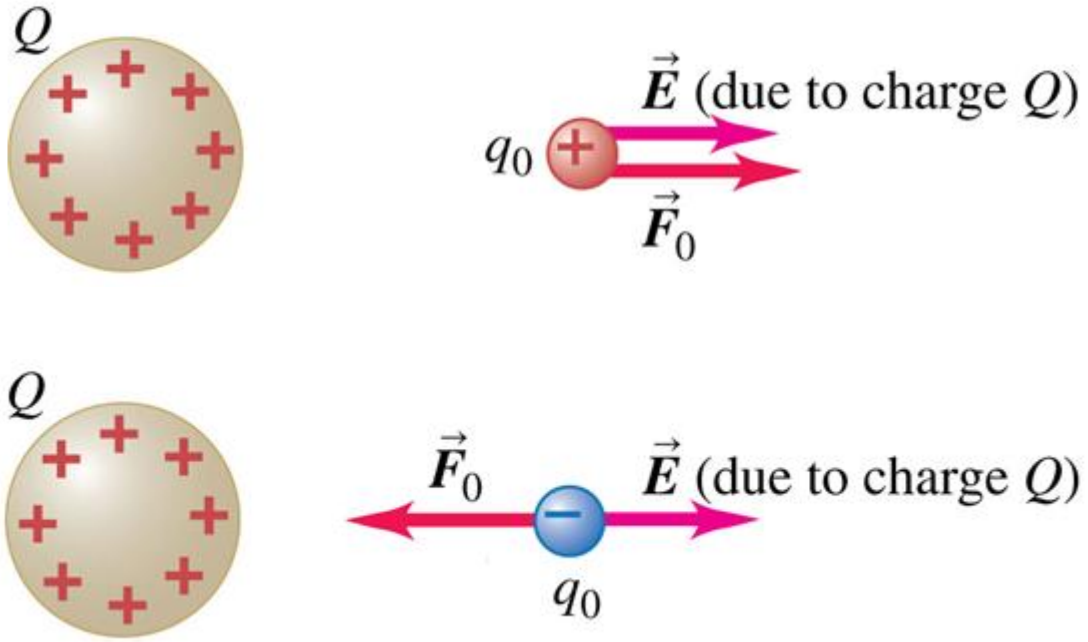
- We can also write the field in terms of the test charge:

$$\boxed{\vec{E} = \frac{\vec{F}_0}{q_0}}$$



Electric field

- Electric field and force can be in opposite directions.



- $\vec{F}_0 = q_0\vec{E}$ is the force exerted on a point charge q_0 by electric field $\vec{E}$.

Comparison to gravity

- Notice the similarities to gravitation:

- Electric force:
  $\vec{\boldsymbol{F}}_0 = q_0\vec{\boldsymbol{E}}$ and
  $\vec{\boldsymbol{F}}_g = m_0\vec{\boldsymbol{g}}$ ……… (Newton's second law equation for object in grav. field)

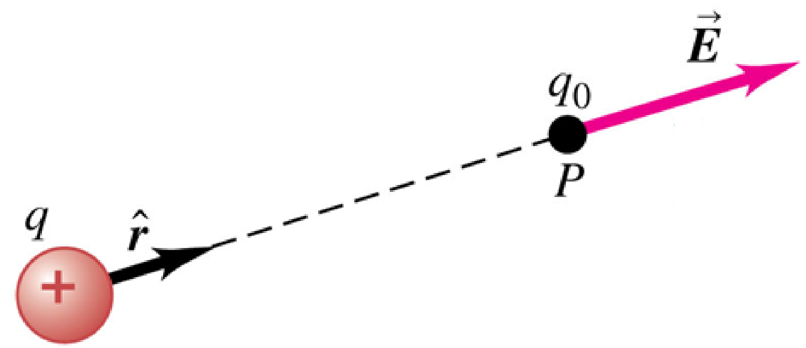
- Electric field:
  $\vec{\boldsymbol{E}} = \vec{\boldsymbol{F}}_0/q_0$ and
  $\vec{\boldsymbol{g}} = \vec{\boldsymbol{F}}_g/m$ ………… (gravitational field from Ch. 13 or "accel. due to grav.")

Units of E: N/C
Recall units of g: m/s^2 = N/kg

Electric field of a point charge

- Consider the positive point charge q and test charge q_0 at P.
- The electric field vector $\vec{\boldsymbol{E}}$ points away from q in direction $\hat{\boldsymbol{r}}$.
- If q had negative charge then $\vec{\boldsymbol{E}}$ would point towards it $(-\hat{\boldsymbol{r}})$.
- The vector equation for the electric field of q at point P is:

$$\boxed{\vec{\boldsymbol{E}}} = \frac{\vec{\boldsymbol{F}}_0}{q_0} = \boxed{k\frac{q}{r^2}\hat{\boldsymbol{r}}}$$

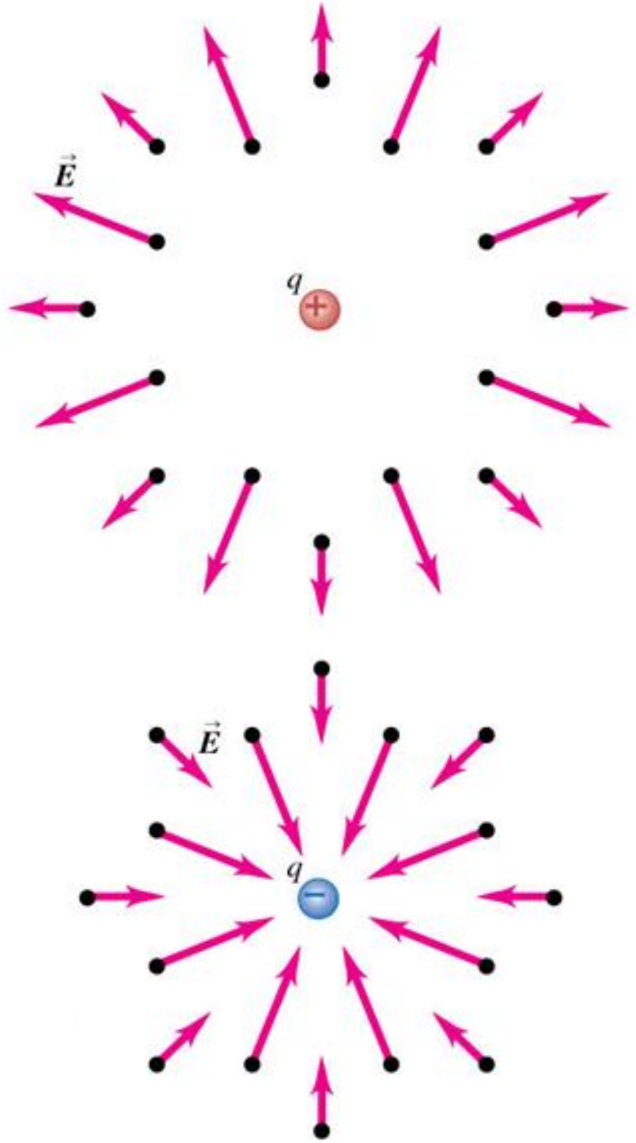


Electric field of a point charge

- By definition, the electric field always points **away** from **positive** charge and *towards negative* charge.

- A point charge produces an electric field at *all* points in space.
  – The **field strength** varies with r^2 and points away in all directions.

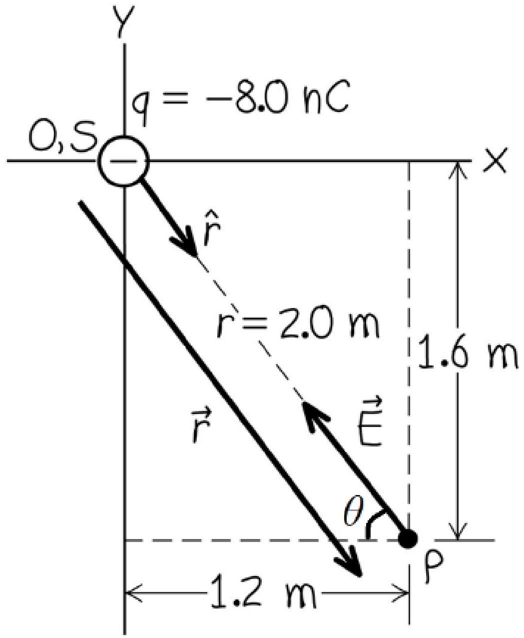
- We can often model objects as point charges (easy if you are at a far away enough distance).



Examples with electric fields

- Ex. **21.5:**
  - What is the magnitude of the electric field $\vec{E}$ at a field point 2.0 m from a point charge $q = 4.0\ \mathrm{nC}$?

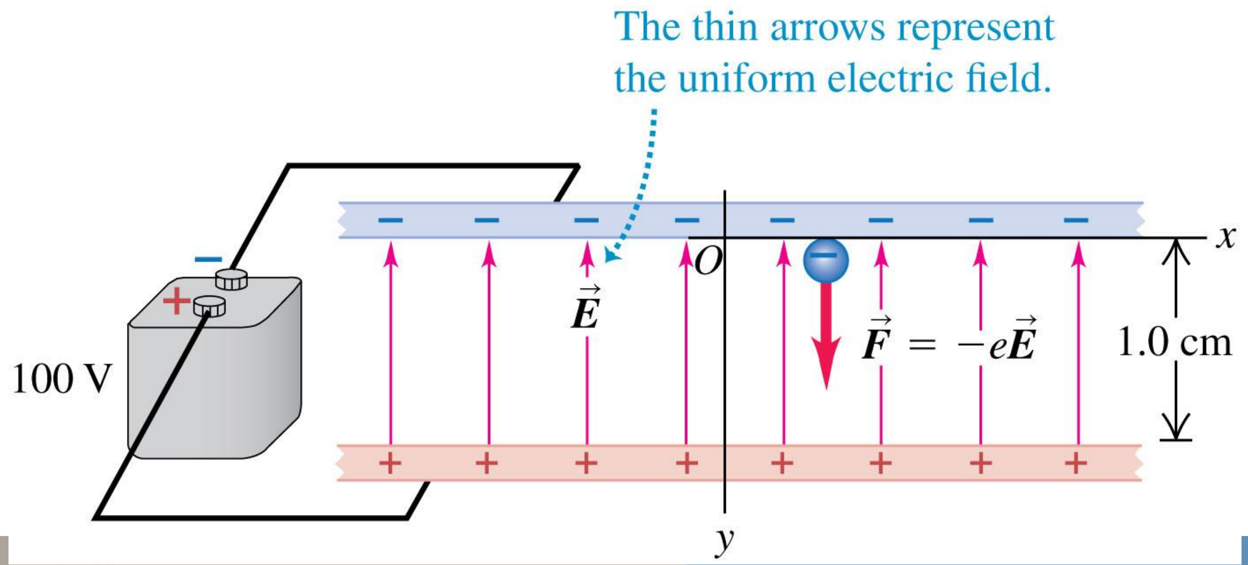
- Ex. **21.6:**
  - A point charge $q = -8.0\ \mathrm{nC}$ is located at the origin. Find the electric field vector at the field point $x = 1.2\ \mathrm{m}$, $y = -1.6\ \mathrm{m}$.



Ex. 21.7

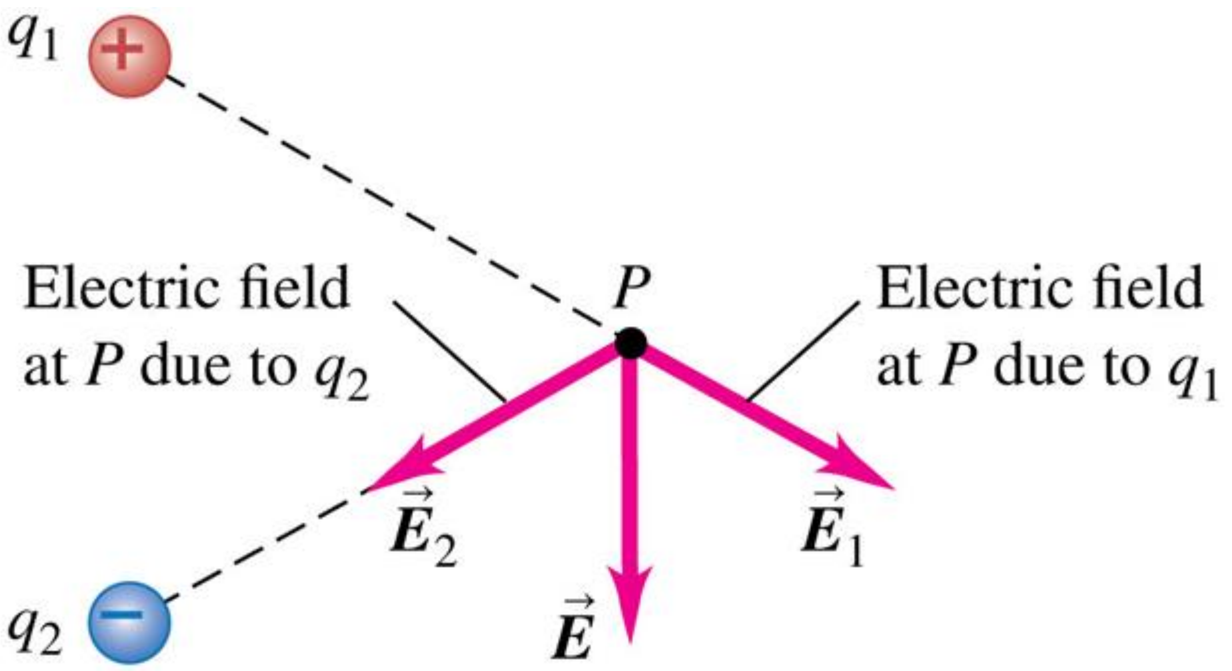
- When the terminals of a battery are connected to two parallel conducting plates with a small gap between them, the resulting charges on the plates produce a *uniform* electric field $\vec{E}$ between the plates. If the plates are 1.0 cm apart and are connected to a 100 volt battery, the field is vertically upward and has magnitude $E = 1.00 \times 10^4$ N/C.

  (a) If an electron is released from rest at the upper plate, what is its acceleration?
  (b) What speed and kinetic energy does it acquire while traveling 1.0 cm to the lower plate?
  (c) How long does it take to travel this distance?



Superposition of fields

- Total E-Field at a point is the vector sum of all fields from present charges.



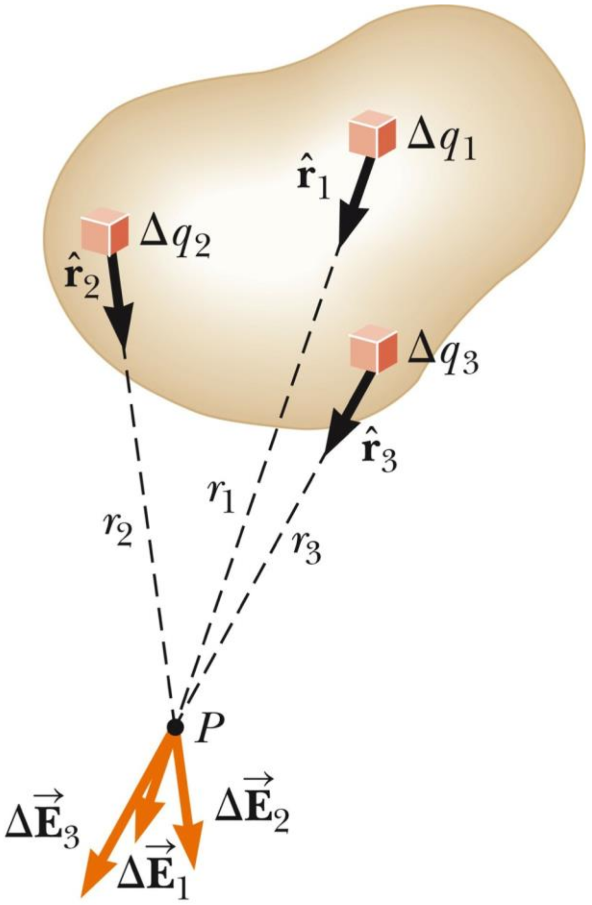
Electric field due to continuous charge

- Divide the charge distribution into small elements with Δq.
- At P, calculate the electric field due to one of the elements.

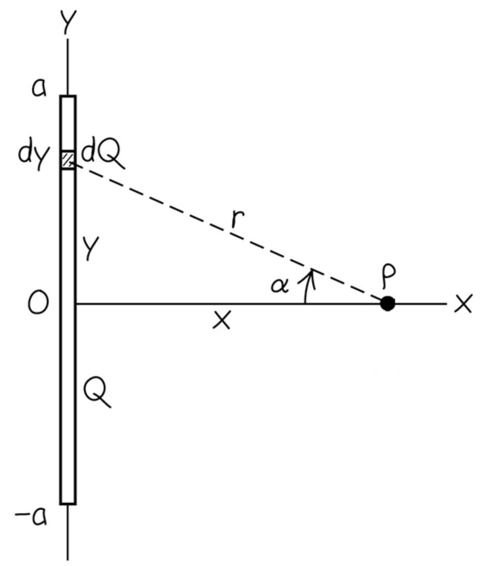
$$\Delta\vec{E} = k\frac{\Delta q}{r^2}\hat{r}$$

- Evaluate the total E-Field by summing up the contribution from all elements:

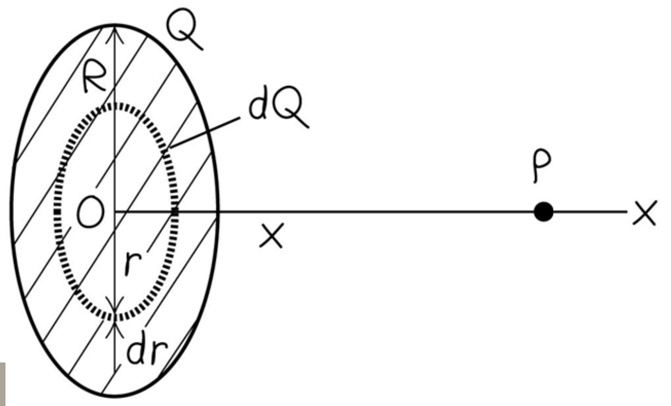
$$\boxed{\vec{E}} = k\lim_{\Delta q_i\to 0}\sum_i \frac{\Delta q_i}{r_i^2}\hat{r}_i = \boxed{k\int\frac{dq}{r^2}\hat{r}}$$

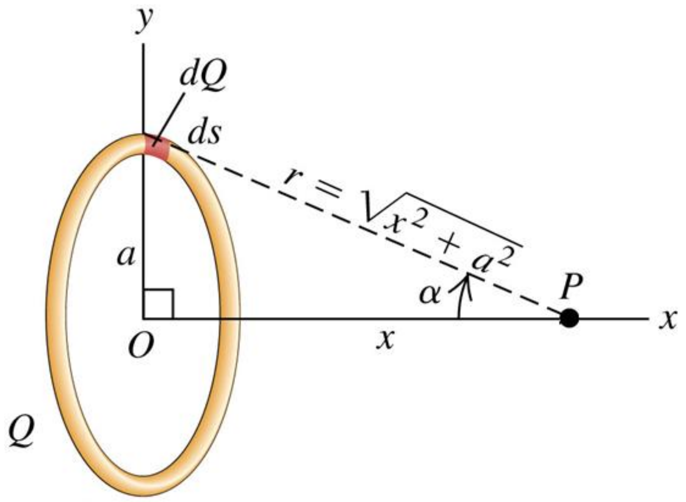


Examples with continuous charge dists.



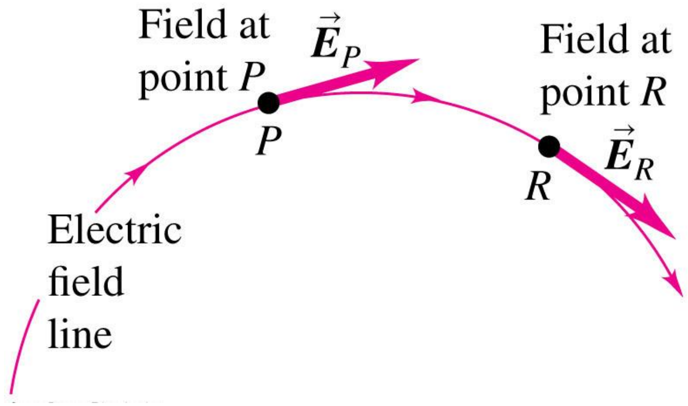
- **Ex. 21.10** – Electric field of a charged line segment.
- **Ex. 21.9** – Field of a ring of charge.
- **Ex. 21.11** – Field of a uniformly charged disk.
- I will post full solutions for these examples.

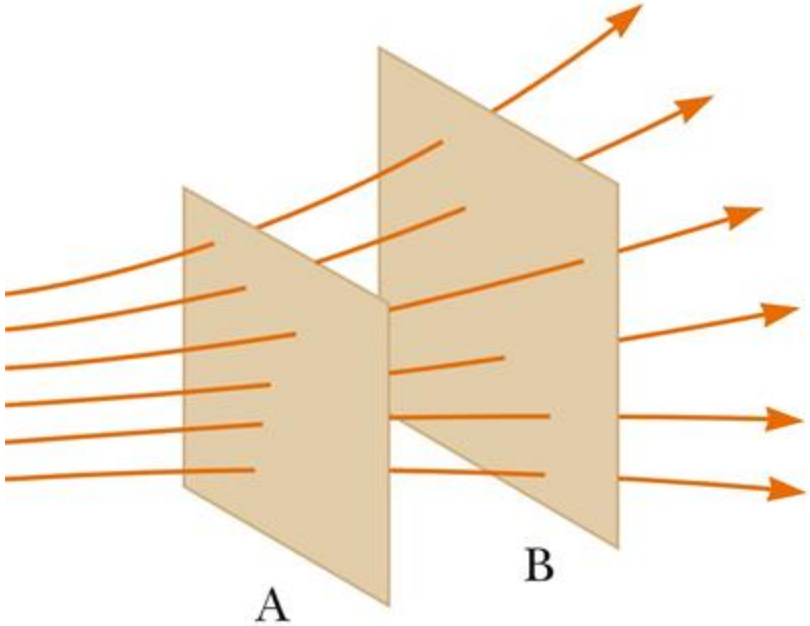




Electric field lines

- Visual representation of the E-Field.
- The E-Field vector, $\vec{E}$, is tangent to the field line at every point.
- We typically draw several lines; the magnitude of the field is higher if the lines are closer together.



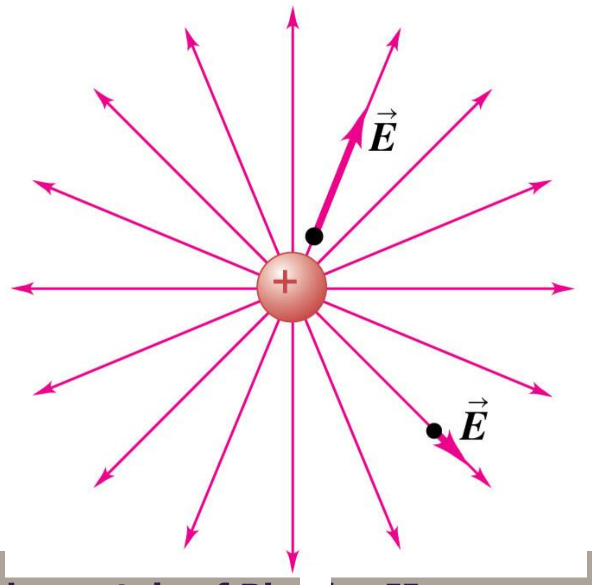


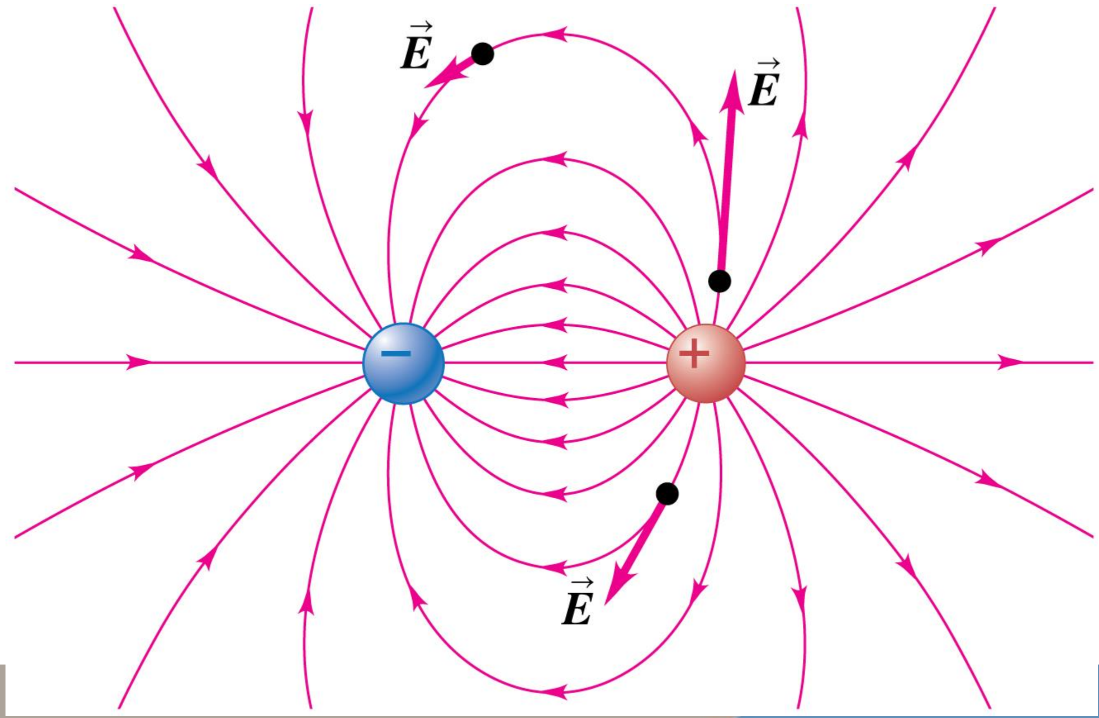
Examples of electric field lines

- Field lines points away from positive, towards negative.
- Number of lines is proportional to magnitude of charge.
- The direction of the field is *tangent* the field line.

- Dipole

- Point Charge

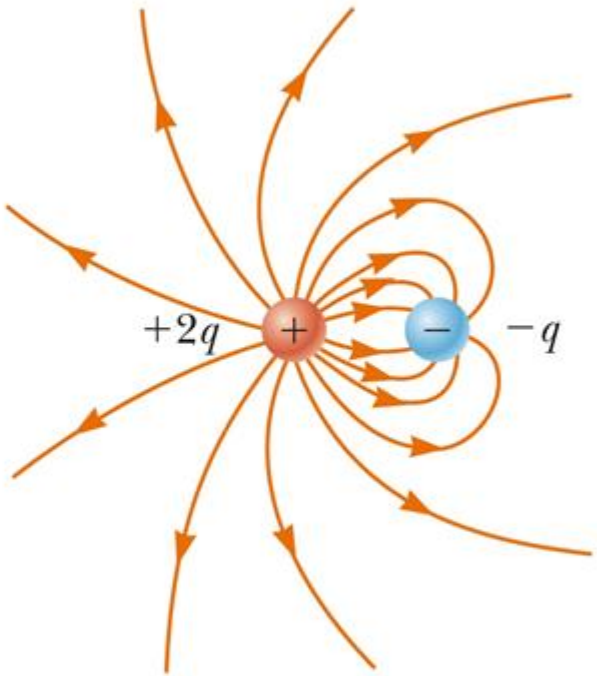




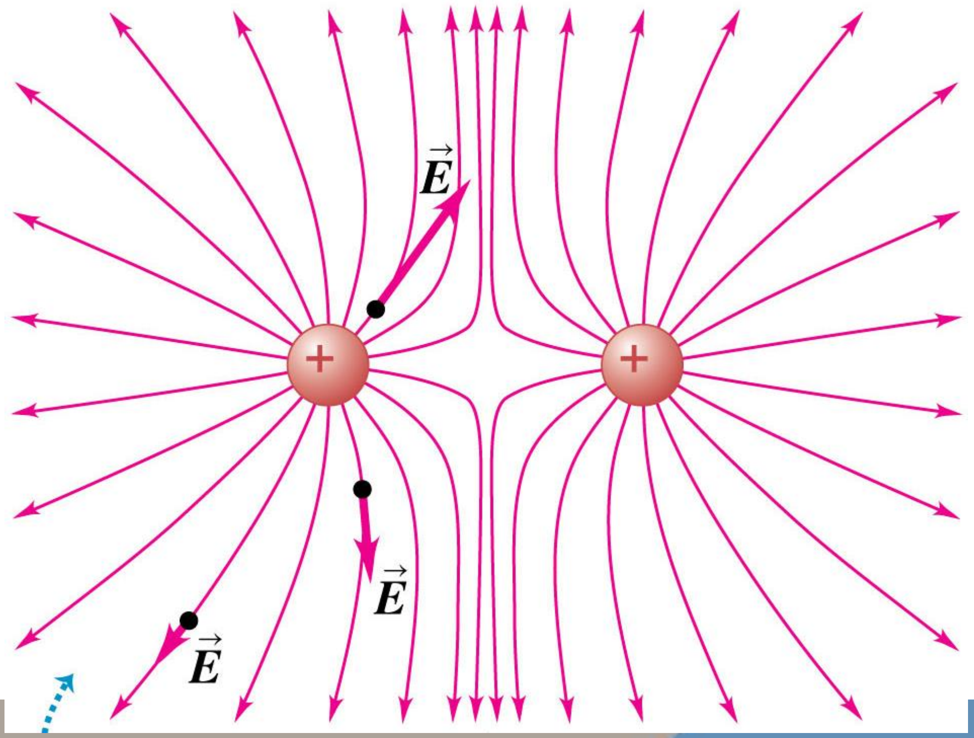
Examples of electric field lines

- Number of lines is proportional to magnitude of charge.
- Field lines never intersect.

- $2q$ and $-q$

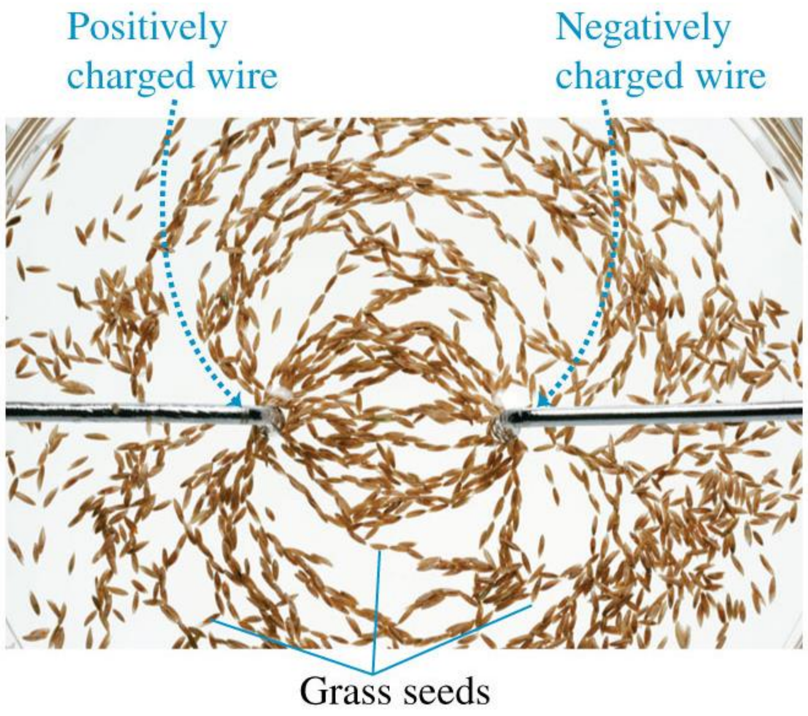


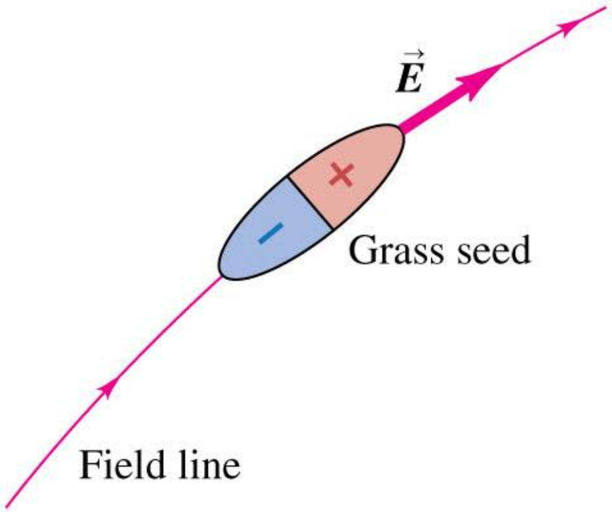
- Two equal positive charges



Demonstration setup of field lines

- Tips of two wires (+ive and –ive) in a liquid with grass seeds floating in it.
- E-Field causes polarization of the seeds so they align with the field.





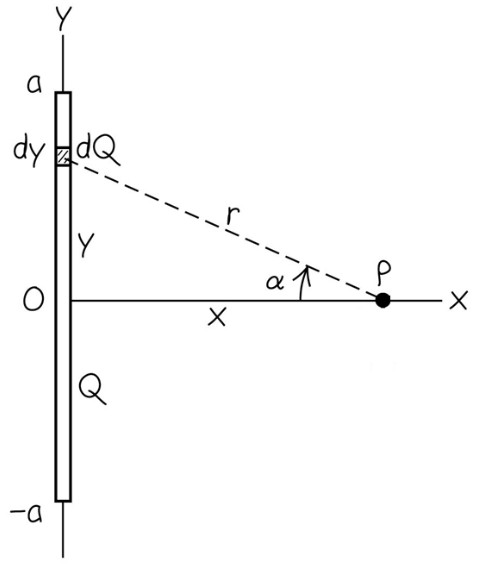
https://www.youtube.com/watch?v=Fhp63yvJAHs

Ex. 21.10 – Electric field of a charged line segment

- Positive charge Q distributed uniformly along the y-axis from $-a$ to a. Find the electric field at point P on the x-axis a distance x from origin.

- Due to symmetry, components in y-axis will cancel, components in x-axis add together:

$$\boxed{E = k\frac{Q}{x\sqrt{x^2 + a^2}}} \text{ direction in } x$$



- If the line was infinitely long (from $y = -\infty$ to $y = +\infty$):

$$\boxed{E = \frac{2k\lambda}{x}} \text{ direction in } x$$

where λ is the linear charge density in units C/m.

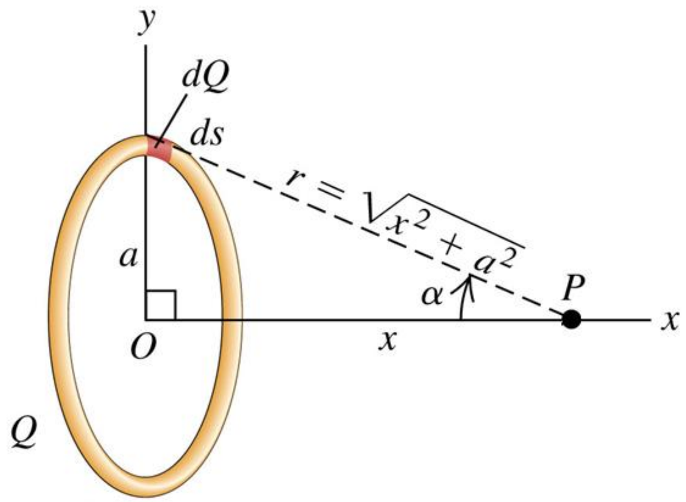
Ex. 21.9 – Field of a ring of charge.

- Charge Q is uniformly distributed around a conducting ring of radius a. Find the electric field at a point P on the ring axis a distance x from the center.

- Due to symmetry, components in y and z axes will cancel, x components add:

$$\boxed{E = k\frac{Qx}{(x^2 + a^2)^{3/2}}} \text{ in direction } x$$

- If $x = 0$ then we are at the center of the ring and $E = 0$.



Ex. 21.11 – Field of a uniformly charged disk

- A disk has radius R with uniform surface charge density σ (in units C/m^2). Find the electric field at a point along the axis of the disk a distance x from its center.

- Solution is to take many concentric ring elements and integrate $r = 0$ to R:

$$\boxed{E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{1}{\sqrt{(R^2/x^2) + 1}}\right)} \text{ in direction } x$$

recall that $\epsilon_0 = 1/4\pi k = 8.854 \times 10^{-12}\ \mathrm{C^2/Nm^2}$.

- If the disk is VERY LARGE ($R \gg x$) then:

$$\boxed{E = \frac{\sigma}{2\epsilon_0}} \text{ in direction } x$$