# Units, vectors, and review

- From Chs. 1-7, 13

Michael Wong – PHY 1122 Spring 2023

# Standards and units

- Different unit systems are used worldwide.
  - CGS (centimeter, gram, second)
  - MKS (meter, kilogram, second)
  - FPS (foot, pound, second …Fahrenheit, yards, miles, leagues, fathoms, ounces, etc… )
  - Atomic units (bohr, electron mass, atomic second)

- We use the **International System (SI)** of units in this course (based on MKS).
  - Metrication in Canada occurred in the 70's.

# Unit prefixes

- Prefixes create larger and smaller units for fundamental quantities:
  - Pico (p) $(10^{-12})$
  - Nano (n) $(10^{-9})$
  - Micro ($\mu$) $(10^{-6})$
  - Milli (m) $(10^{-3})$
  - Centi $(10^{-2})$

  - Kilo (k) $(10^{3})$
  - Mega (M) $(10^{6})$
  - Giga (G) $(10^{9})$
  - Tera (T) $(10^{12})$

Examples we'll see:

Capacitance measured in nF or $\mu$F.
Charge measured in mC or $\mu$C.

Time for light to travel down
a 1 m hollow tube is 3.333 ns.

Truck engine burns gasoline and
generates kJ or MJ of heat energy.

# Unit consistency and conversions

- Your equations must be dimensionally consistent.
  - Make sure you're adding apples to apples.
    Not apples to bananas or grapefruits.

- Convert units by forming ratios of same physical quantity in two different units:

  - Eg. Find the number of seconds in 3 minutes

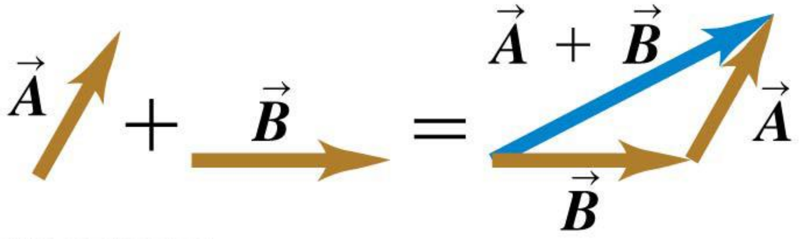
$$3\ \mathrm{min} = (3\ \cancel{\mathrm{min}})\left(\frac{60\ \mathrm{s}}{1\ \cancel{\mathrm{min}}}\right) = \boxed{180\ \mathrm{s}}$$

# Scalars and vectors

- A **scalar quantity** can be described by a *single* number and combine according to usual arithmetic rules.

- A **vector quantity** has both a *magnitude* and *direction* in space.
  - Use bold with arrow above to represent a vector: $\vec{\boldsymbol{R}}$
  - The magnitude is $R = |\vec{\boldsymbol{R}}|$

- Use addition/subtraction to combine vectors.



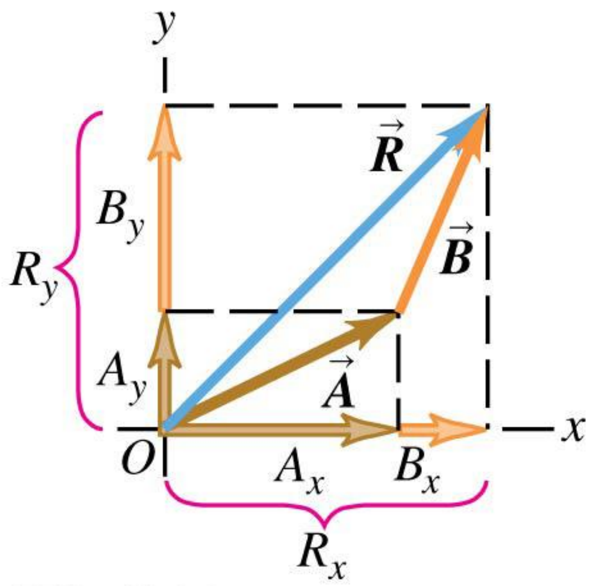
# Vector components and unit vectors

- Vectors can be added by using (cartesian) coordinates.
  - It helps to break vectors into **components**:

$$\vec{\boldsymbol{A}} = \left(A_x, A_y\right)$$

  - The $x$-component of $\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$ is the sum of the individual $x$-components of $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ and similarly for $y$.

$$\boxed{R_x = A_x + B_x}\ ,\ \boxed{R_y = A_y + B_y}$$




- **Unit vectors** describe directions in space.
  - Has magnitude of 1 and no units.

$$\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}}$$

# Adding two right angle vectors

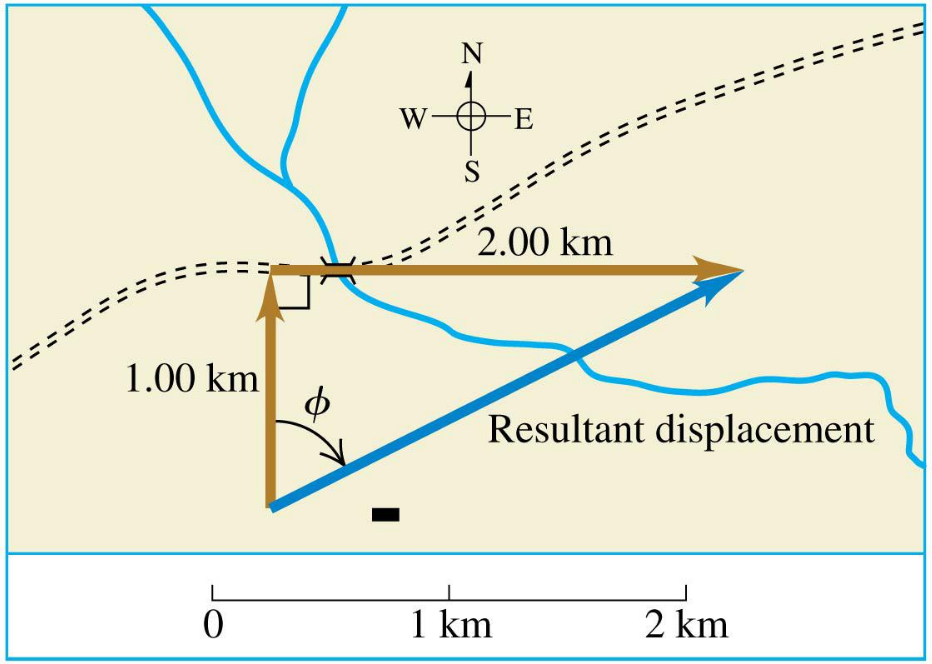
- Use trigonometry to find magnitude and direction of resulting displacement.

- Pythagoras:

$$R = \sqrt{(1\text{ km})^2 + (2\text{ km})^2}$$
$$\boxed{R = 2.236\text{ km}}$$

Trigonometry:

$$\cos\phi = \frac{1\text{ km}}{2.236\text{ km}}$$
$$\boxed{\phi = 63.4°}$$



# Scalar product

- The scalar product (dot product) of two vectors is a scalar:

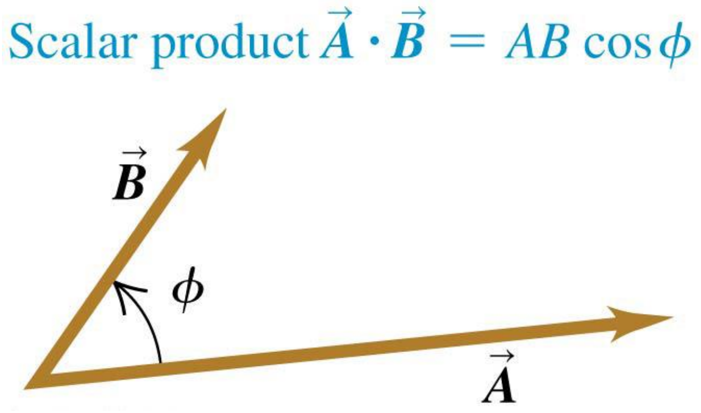
$$C = \vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = \boxed{AB\cos\phi} = \boxed{A_x B_x + A_y B_y}$$

  where $\theta$ is the angle between the vectors.

- Scalar product is commutative: $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}}$
- Also distributive: $\vec{\boldsymbol{A}} \cdot \left(\vec{\boldsymbol{B}} + \vec{\boldsymbol{C}}\right) = \vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} + \vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{C}}$

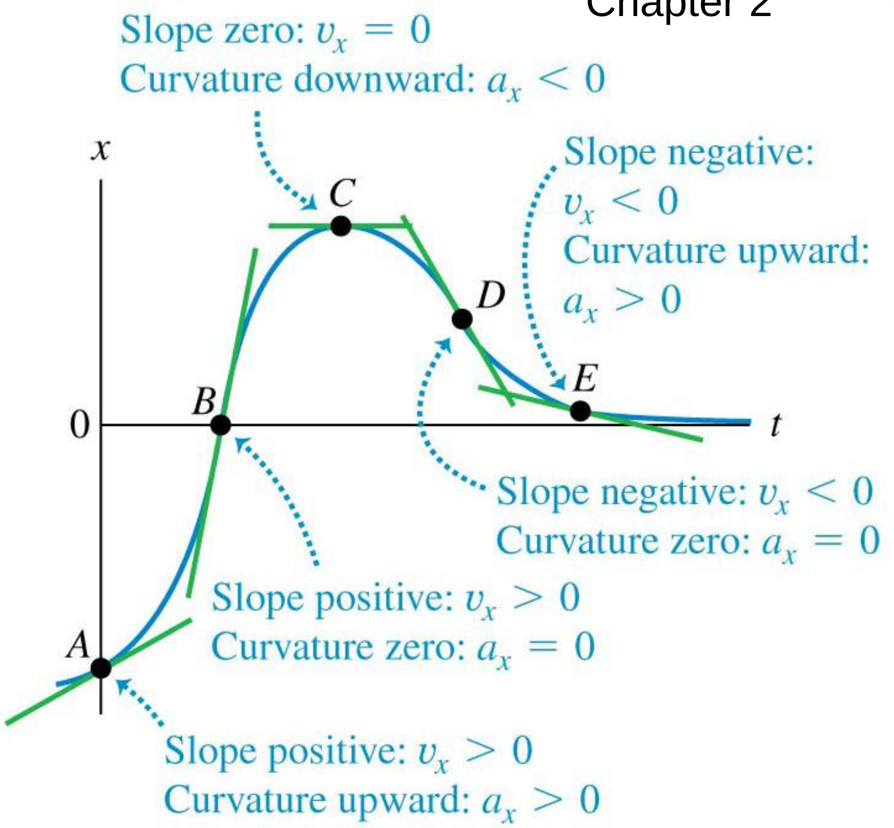
- For unit vectors:

$$\hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\imath}} = \hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{\jmath}} = 1$$
$$\hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\jmath}} = \hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{\imath}} = 0$$



# Motion in 1D

- In 1D, a particle has
  - **position** $x$
  - displacement $\Delta x = x_2 - x_1$
  - average velocity $v_{avg} = \Delta x / \Delta t$
  - instantaneous **velocity** $v_x = dx/dt$
  - average acceleration $a_{avg} = (v_2 - v_1)/\Delta t = \Delta v_x / \Delta t$
  - instantaneous **acceleration** $a_x = dv_x/dt = d^2x/dt^2$




- Model: Particle under constant velocity (or speed): $\boxed{x = x_0 + v_x t}$

- Model: Particle under constant acceleration:

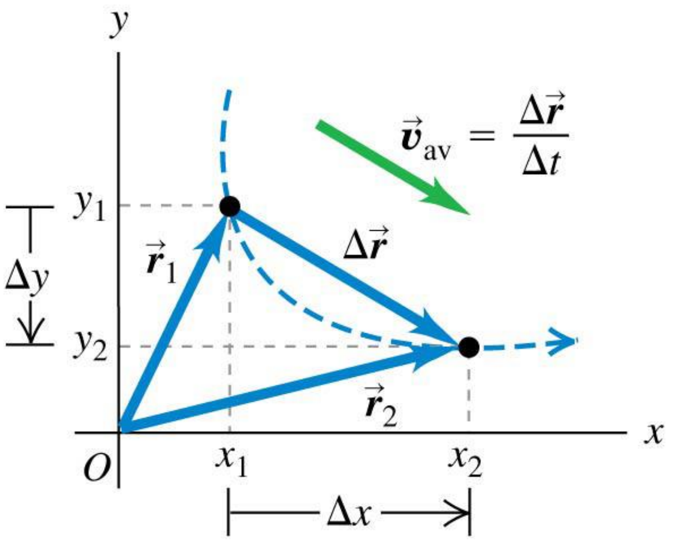
$$\boxed{v = v_0 + at}\ ,\ \boxed{x - x_0 = \frac{1}{2}(v_0 + v_x)t}\ ,\ \boxed{x = x_0 + v_0 t + \frac{1}{2}a_x t^2}\ ,\ \boxed{v^2 = v_0^2 + 2a_x(x - x_0)}$$

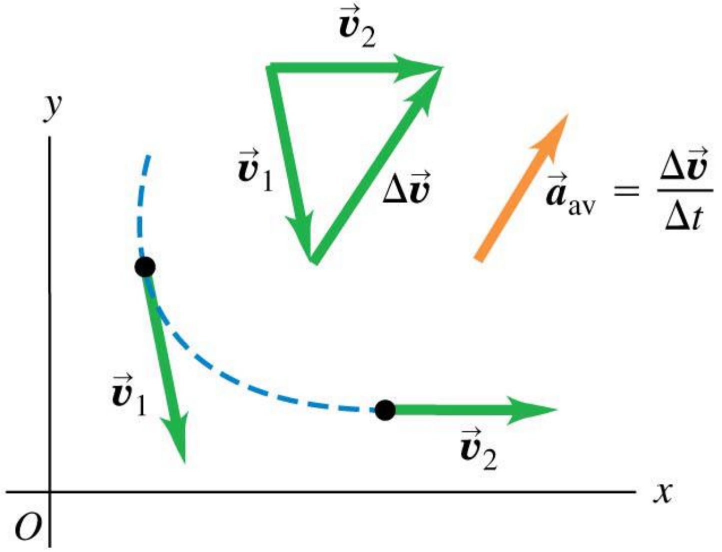
# Motion in 2D

- Must represent properties of motion as vectors:
  - Position $\vec{\boldsymbol{r}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$
  - Velocity $\vec{\boldsymbol{v}} = d\vec{\boldsymbol{r}}/dt$
  - Acceleration $\vec{\boldsymbol{a}} = d\vec{\boldsymbol{v}}/dt = d^2\vec{\boldsymbol{r}}/dt^2$

- Equations of motion:
  - Model: object under constant acceleration

$$\vec{\boldsymbol{r}}_f = \vec{\boldsymbol{r}}_i + \vec{\boldsymbol{v}}_i t + \frac{1}{2}\vec{\boldsymbol{a}}t^2$$
$$\vec{\boldsymbol{v}}_{\boldsymbol{f}} = \vec{\boldsymbol{v}}_{\boldsymbol{i}} + \vec{\boldsymbol{a}}\boldsymbol{t}$$



# Types of forces

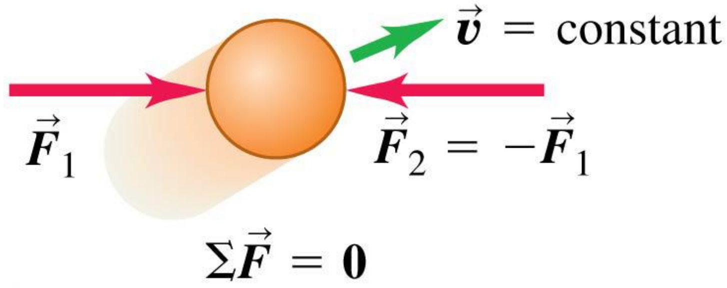
- In physics: **force** is quantitative measure of interaction between two objects.
- We have **contact forces** such as pushing, pulling, etc…
- We also have **field forces** – no contact, represented by the 4 fundamental forces of nature:
  - 1) gravitational force
  - 2) **electromagnetic** force
  - 3) strong force
  - 4) weak force

# Newton's laws of motion

- **Force** is a vector quantity. When multiple forces act on an object, the effect on motion is same as a *single* force that is equal to vector *sum of all forces*:

$$\Sigma\vec{\boldsymbol{F}} = \vec{\boldsymbol{F}}_1 + \vec{\boldsymbol{F}}_2 + \cdots$$

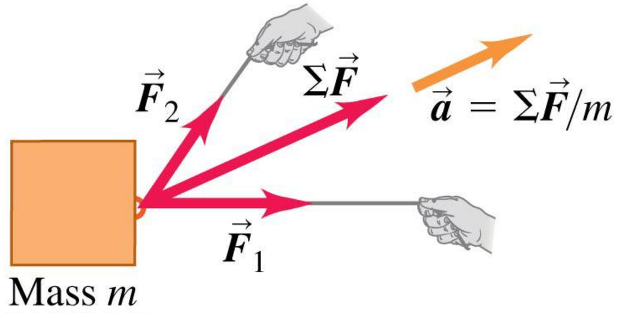
- **Newton's 1st law:**
  - If the net force on an object is zero ($\Sigma\vec{\boldsymbol{F}} = 0$) then the body is in equilibrium ($\vec{\boldsymbol{a}} = 0$).



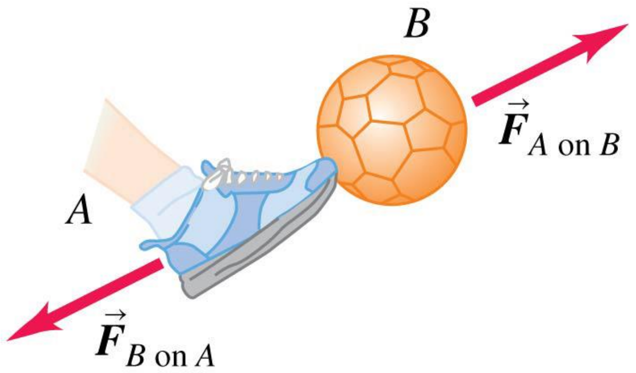
# Newton's laws of motion

- **Newton's 2nd law:**
  - **Inertia** of an object is related to its mass.
  - Acceleration of an object is proportional to force and is inversely proportional to mass.

$$\Sigma\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$$




- **Newton's 3rd law:**
  - When two objects interact, they exert forces on each other that are equal in magnitude and opposite in direction.

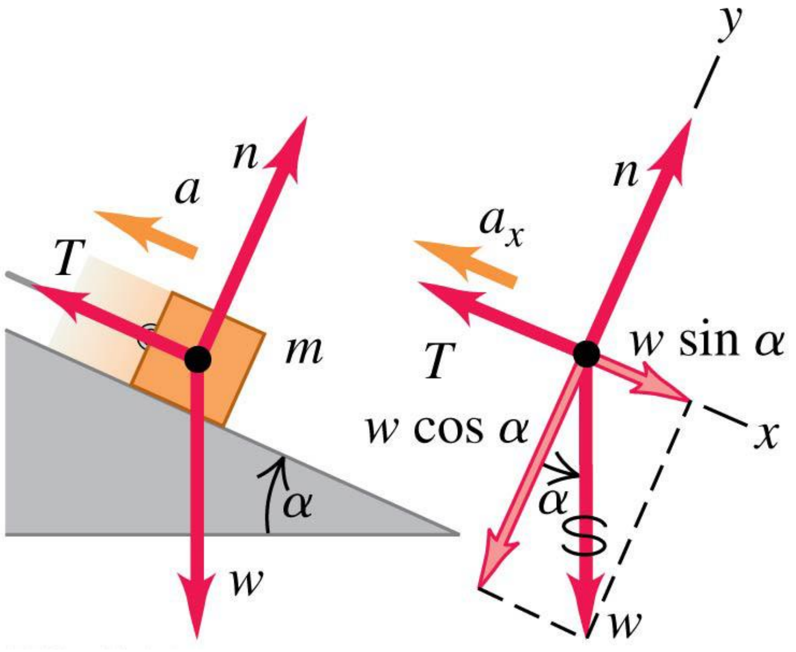


# Breaking down force components

- Consider the block in equilibrium attached to a string on the inclined plane at angle $\alpha$.
  - We can break down the force of gravity ($w = mg$) into two components:

    $x$-direction:
    $w \sin\alpha$ down the plane (counteracts the tension, $T$)

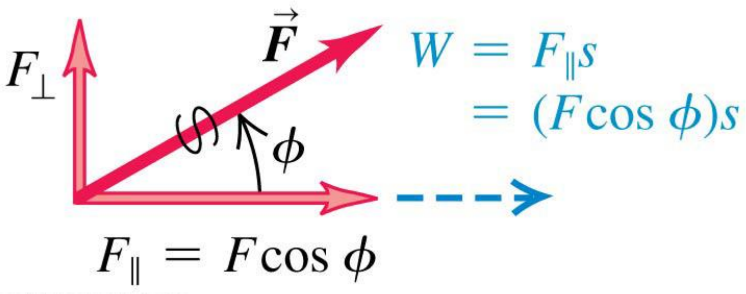
    $y$-direction:
    $w \cos\alpha$ into the plane (balances the normal force, $n$)



# Work done by a force

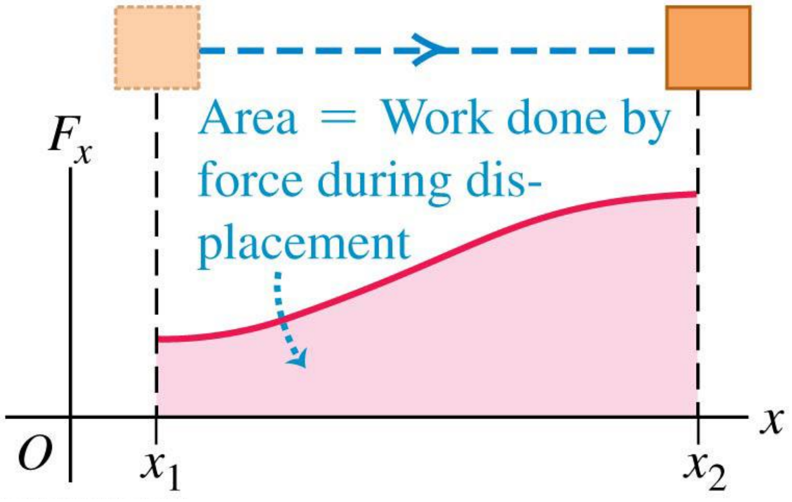
- When a constant force $\vec{F}$ acts on a particle that moves in a straight line with displacement $\vec{s}$, the **work** done (measured in J) by the force is:

$$W = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{s}} = Fs\cos\phi$$




- If the force varies over time or if the movement is curved,

$$W = \int_1^2 \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}}$$



# Work, kinetic energy, and power

- The **kinetic energy** of a particle (also in joules) is a measure of work done to accelerate a particle from rest to speed $v$:

$$K = \frac{1}{2}mv^2$$

- The **work-kinetic energy theorem** tells us that when forces act on an object and displacement occurs, the change in kinetic energy is equal to the amount of work done.

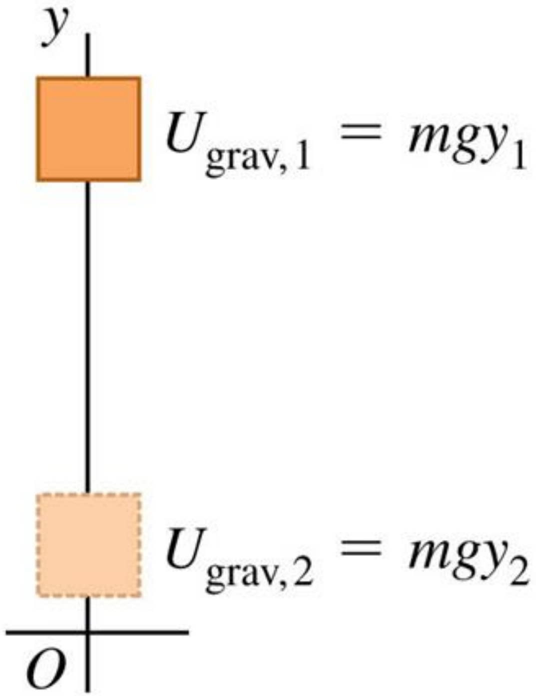
- **Power** (measured in W) is the time rate of doing work.
  - Average power: $P_{\mathrm{av}} = \Delta W / \Delta t \qquad \mathrm{J/s} = \mathrm{W}$
  - Instant power: $P = dW/dt = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{v}}$

# Work and potential energy

- Shared energy between two (or more) objects in a system with lack of motion is represented by potential energy $U$.

- The work done by constant gravitational force $F_g = mg$ leads to change in **gravitational potential energy**:

$$W_{\text{grav}} = mgy_1 - mgy_2 = -\Delta U_{\text{grav}}$$



# Conservation of energy

- A **conservative force** is one where the work-kinetic energy relationship is reversible. Otherwise, it is called a **non-conservative force** (like friction) and leads to changes in internal energy.

- The sum of kinetic, potential, and internal energies in a closed system is always conserved:

$$\Delta K + \Delta U + \Delta U_{\text{int}} = 0$$

- If there is no internal energy, we often use:

$$\boxed{K_1 + U_1 = K_2 + U_2}$$

# Newton's law of gravitation

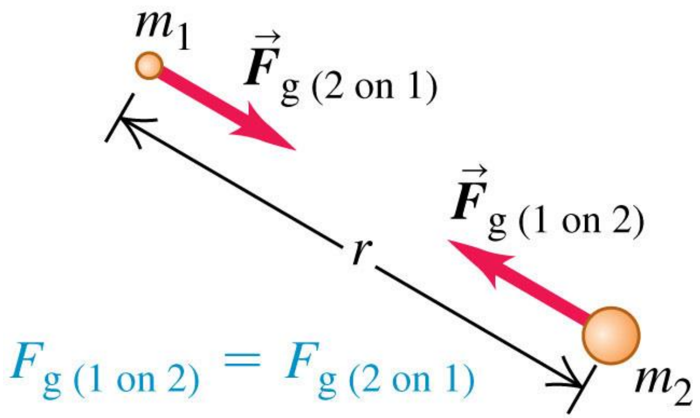
- *Any* two particles with mass attract each other with force:

$$F_g = G\frac{m_1 m_2}{r^2}$$

  where $G = 6.67408 \times 10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg{\cdot}s}^2}$ is the gravitational constant.

- Gravitational force is an **inverse square law**.

- Any object with mass generates a **gravitational field** around it that applies the force $F_g$ on an object that comes near it.



# Gravitational force, weight, and energy

- The **weight** of an object is the total gravitational force exerted on it by all other bodies. On the surface of the Earth, the force is $F_g$.
  - Because the Earth is so massive, its force dominates:

$$w = F_g = G\frac{m_E m}{R_E^2} \quad \rightarrow \quad g = \frac{F_g}{m} = G\frac{m_E}{R_E^2} \approx 9.8\ \mathrm{m/s^2}$$

- For any object-Earth system, the **gravitational potential energy** is inversely proportional to the separation $r$:

$$U = -G\frac{m_E m}{r}$$

- These concepts of **force**, **field**, and **potential energy** have exact analogues in **electromagnetism**.