Chapter 26

- Direct-Current (DC) Circuits

Michael Wong – PHY 1122 Spring 2023

Learning goals

- Resistors in **series** and **parallel**
- **Kirchhoff's rules** for circuit analysis
- Measuring current, voltage, resistance in a circuit
- Analyzing **RC circuits**.

Electric circuit analysis

- We will learn general methods for analyzing complex networks of circuit elements
  - Batteries, resistors, capacitors, etc…
  - Kirchhoff's rules (based on conservation of charge, energy)

- Resistors in series and parallel – similar (but opposite) rules compared to capacitors.

- We analyze circuits with DC (direct current)

DC vs AC

- **Direct Current** (DC) in a circuit has constant direction.
  - We analyze circuits where things are in *steady state* (constant magnitude and direction of current).

- A battery produces DC voltage, household wall sockets provide alternating current (AC) voltage.
  - Adapters convert AC to DC to power your phone, laptop, *sewing machine*, etc…

Resistors in series

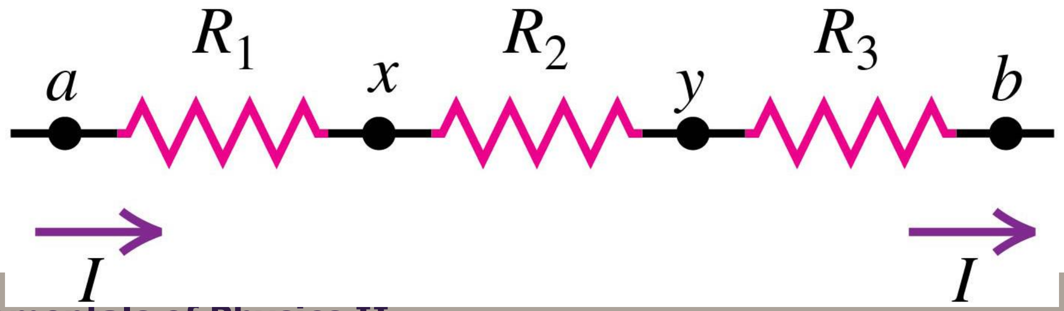
- Resistors in **series** are connected one after the other, current is the same:

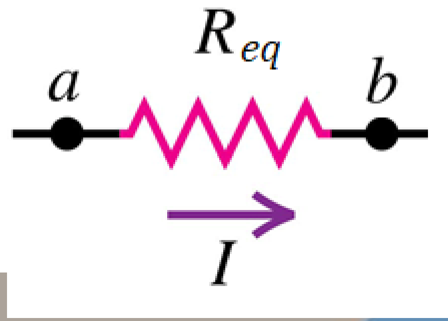
$$I = I_1 = I_2$$

- The voltage adds up:

$$V_{ab} = V_{ax} + V_{xy} + V_{yb}$$
$$V_{ab} = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3) = IR_{eq}$$

- We see that the equivalent resistance is: $\boxed{R_{eq} = R_1 + R_2 + \cdots}$



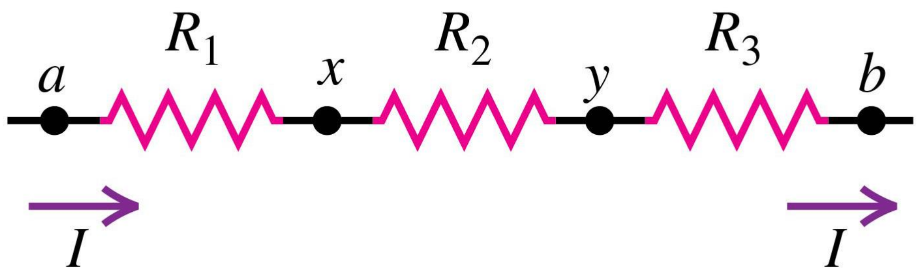


Resistors in series (notes)

- If one device in a series circuit fails, all devices will stop working (cheap old Christmas lights).

- A local change in one part may result in a global change throughout.
  - Change R_1 and the voltage in all other resistors and the terminal voltage of battery will change.

- In series circuit, only one path for current to take.
  - Hence why $I = I_1 = I_2 = \cdots$



Resistors in parallel

- For resistors in **parallel**, potential difference between each is the same:

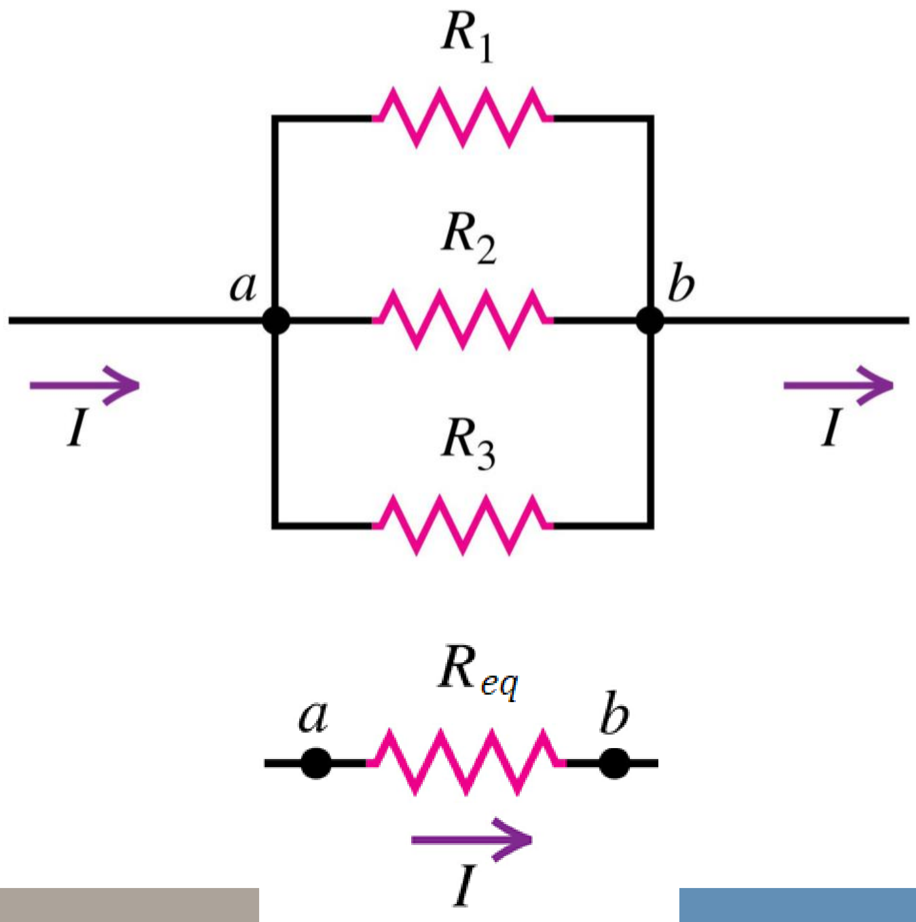
$$V_{ab} = V_1 = V_2 = V_3$$

- The current is summed:

$$I = I_1 + I_2 + I_3$$
$$I = \left(\frac{V_1}{R_1}\right) + \left(\frac{V_2}{R_2}\right) + \left(\frac{V_3}{R_3}\right)$$
$$I = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = V_{ab}\frac{1}{R_{eq}}$$

- The equivalent resistance is thus:

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots}$$

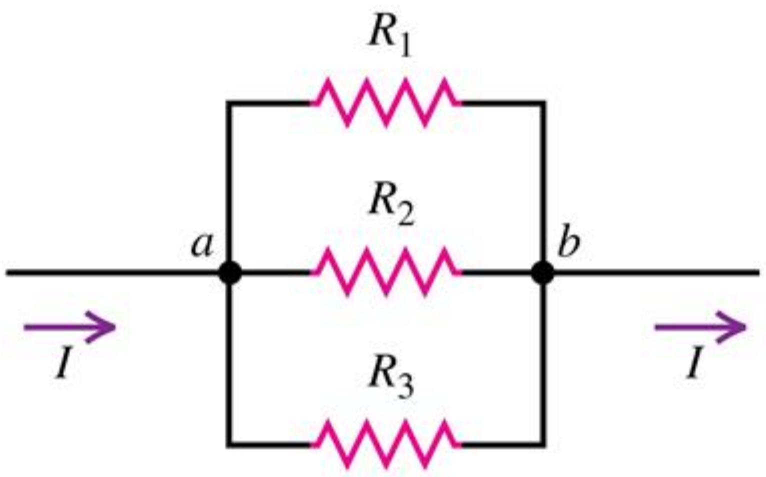


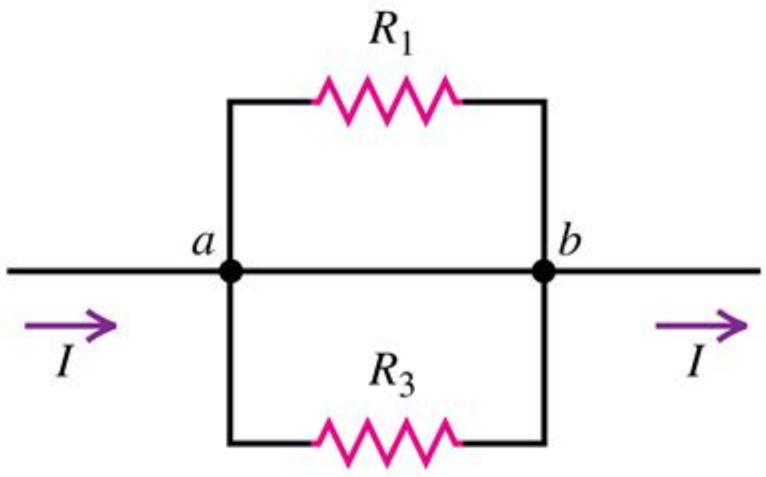
Resistors in parallel (notes)

- Each device operates independently of the others → if one switched off, others remain on

- The current takes all paths in a circuit.
  - Lower R has higher I (path of least resistance)
    - Ie. Short circuit as shown in the bottom pic.

- Household circuits typically wired so that electrical devices are connected in parallel.



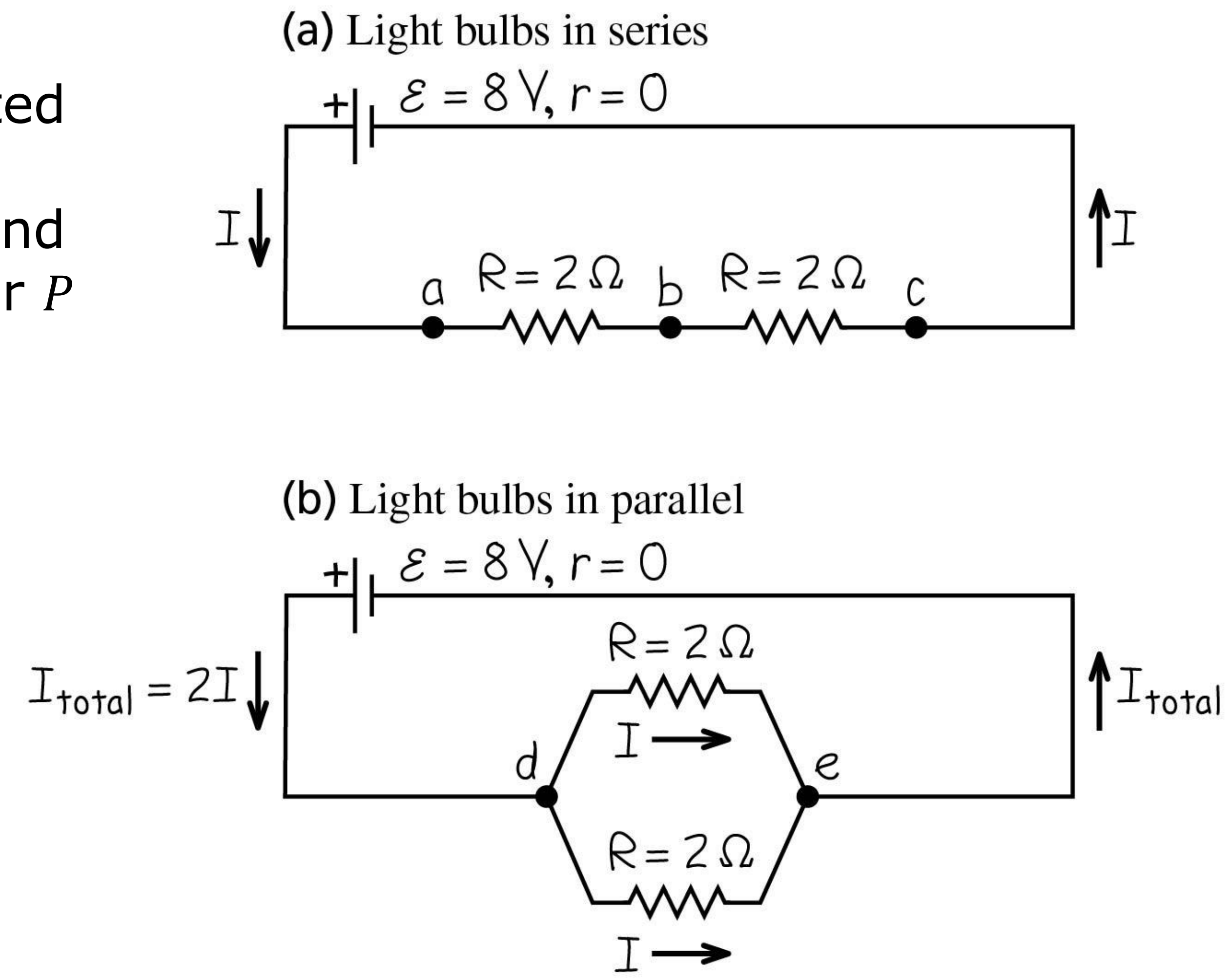


Ex. 26.2 – Series vs parallel

- Two light each with $R = 2\,\Omega$ are connected to source with $\varepsilon = 8\,\mathrm{V}$ and negligible internal resistance. Find the current I and voltage V through each bulb, and power P delivered for:

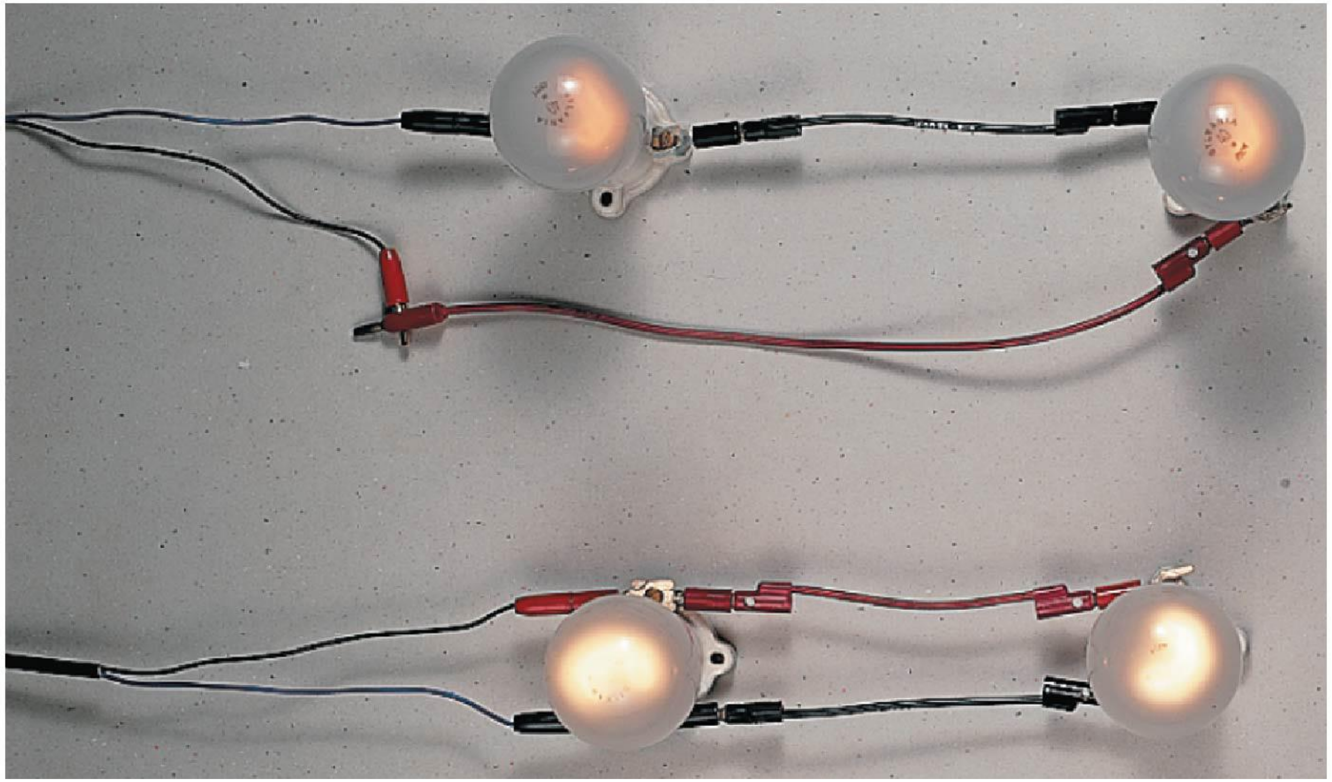
  (a) series connection
  (b) parallel connection

  (c) Suppose one bulb burns out, what happens in each case (series vs parallel)?



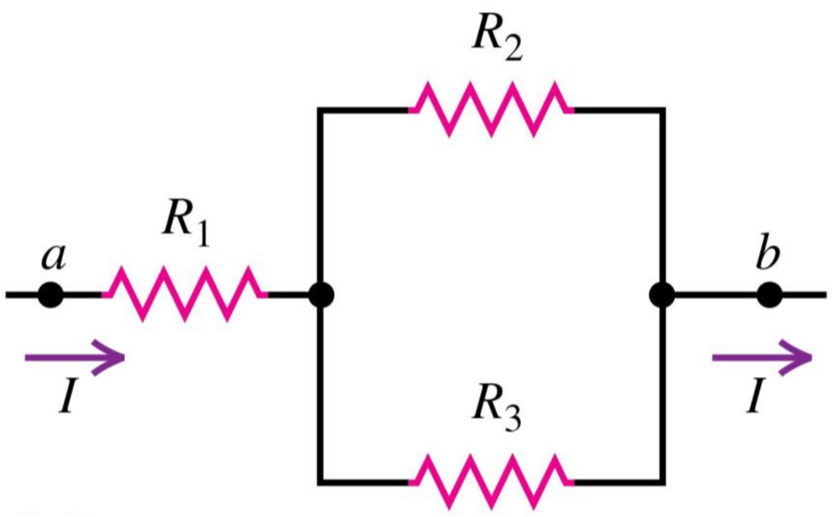
Ex. 26.2 – Series vs parallel (cont.)

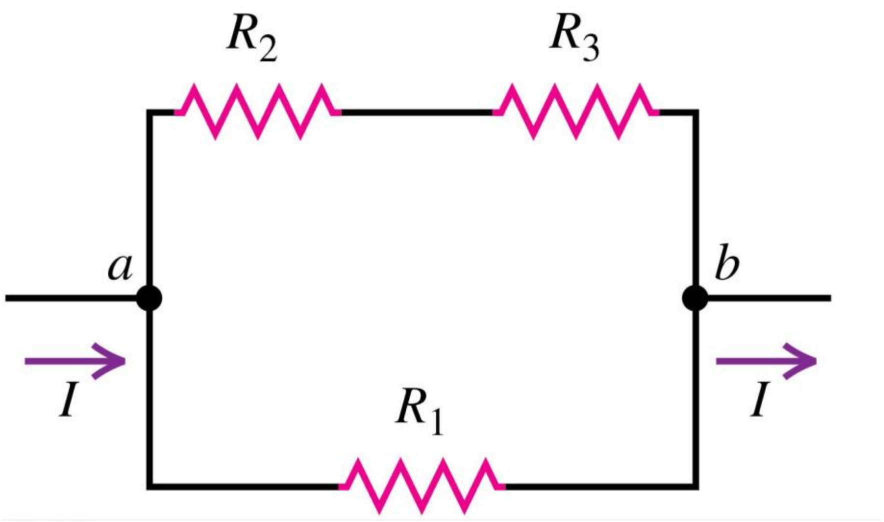
- When connected to same source, two bulbs in series (top) draw less power and are dimmer compared to parallel connection (below).



Series and parallel combinations

- We can reduce these simple combinations:



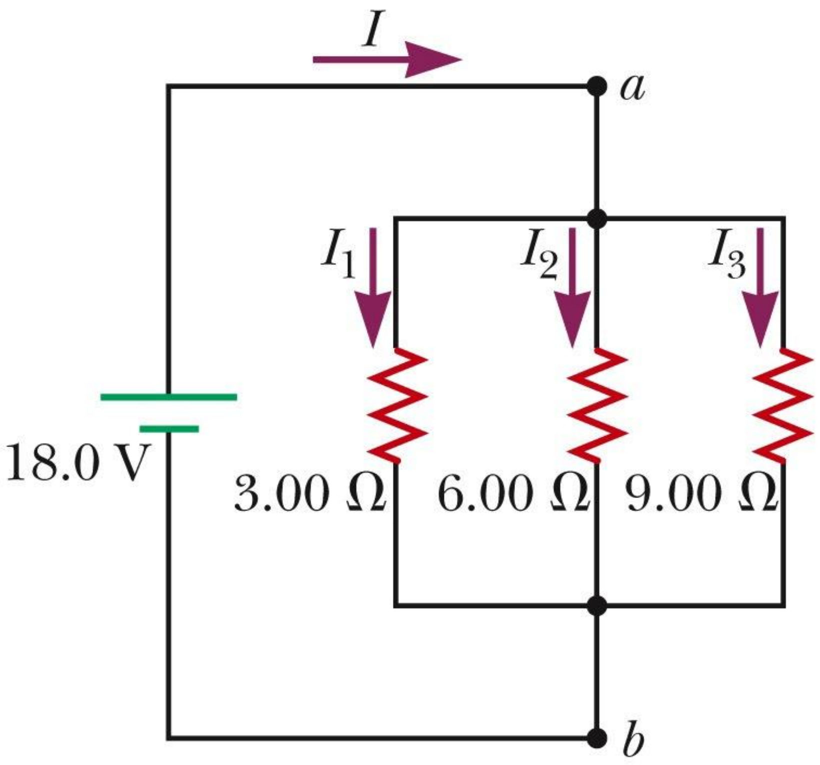


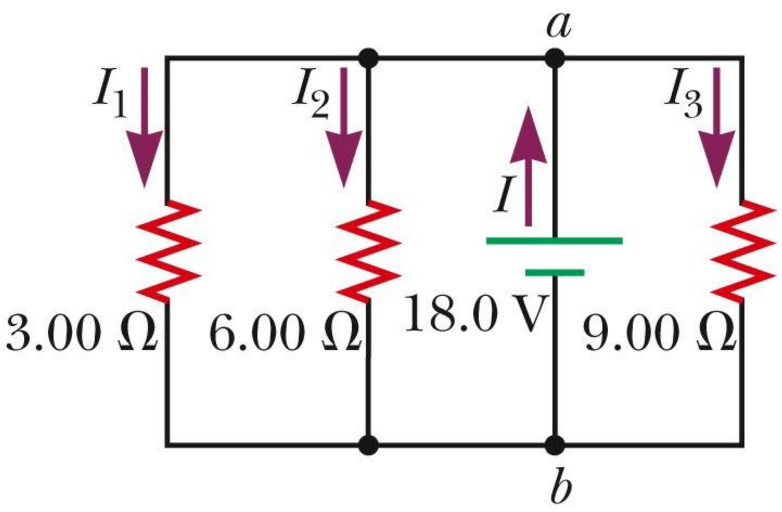
$$R_{eq1} = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}}$$

$$R_{eq2} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + R_3}}$$

Tricky circuit drawings…

- Are the two circuits below equivalent?

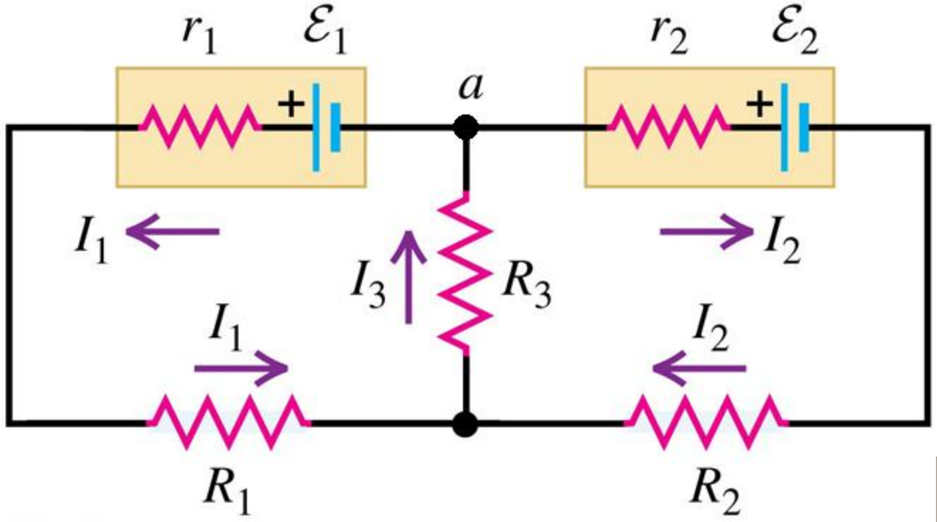




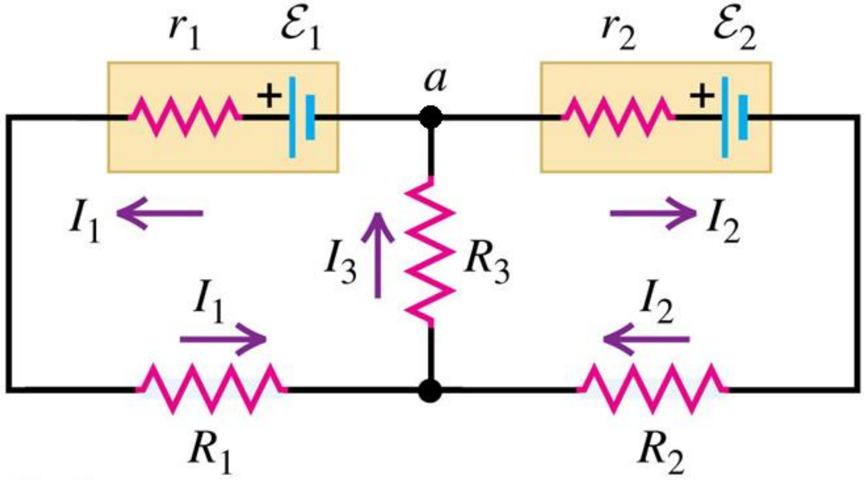
Kirchhoff's rules for circuit analysis

- Sometimes we have a complicated circuit which we can't reduce down to a single R_{eq}.
  - Ie. Circuit with multiple voltage sources.

- We use two rules, called **Kirchhoff's Rules**, to analyze.
  - Based on conservation of charge (**Junction rule**) and conservation of energy (**Loop rule**).



Kirchhoff's rules – Junction



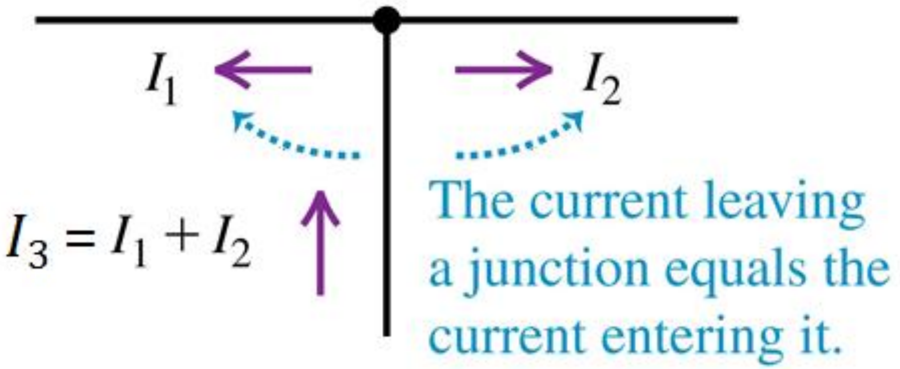
- A **junction** is a point where three or more wires (conductors) meet.

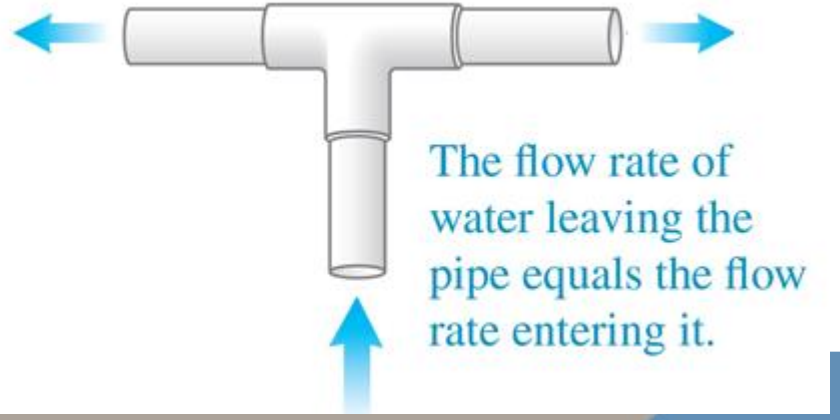
- The sum of the currents at any junction equals zero.
  - Current directed into the junction is positive $(+I)$
    Current leaving the junction is negative $(-I)$

$$\boxed{\sum_{\text{junction}} I = 0}$$

At a, $0 = I_3 - I_1 - I_2$

$$\boxed{I_3 = I_1 + I_2}$$



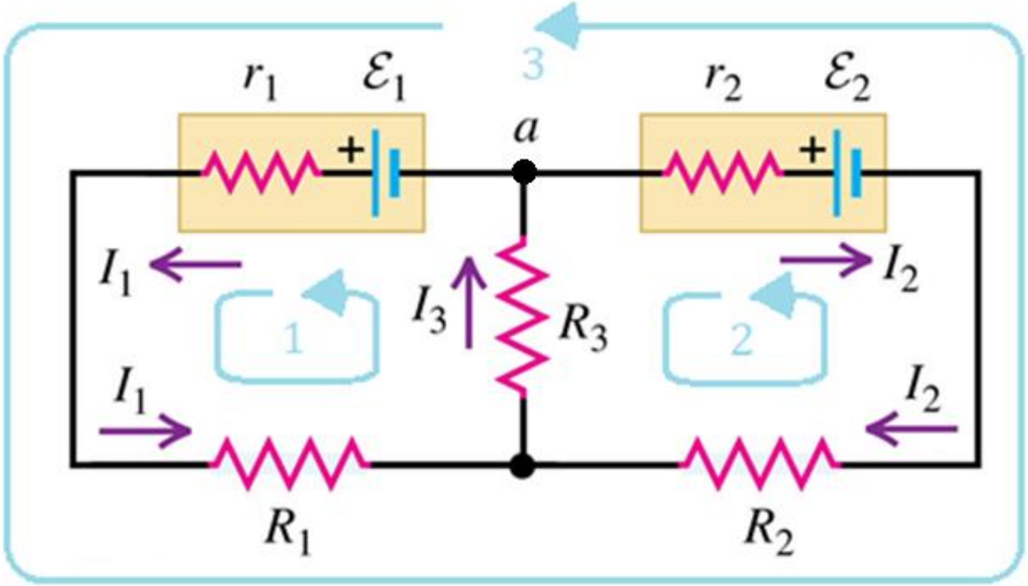


Kirchhoff's rules – Loop

- A **loop** is any closed conducting path.

- The sum of the potential differences across all elements around any closed loop must be zero,

$$\boxed{\sum_{\substack{\text{closed}\\\text{loop}}} \Delta V = 0}$$



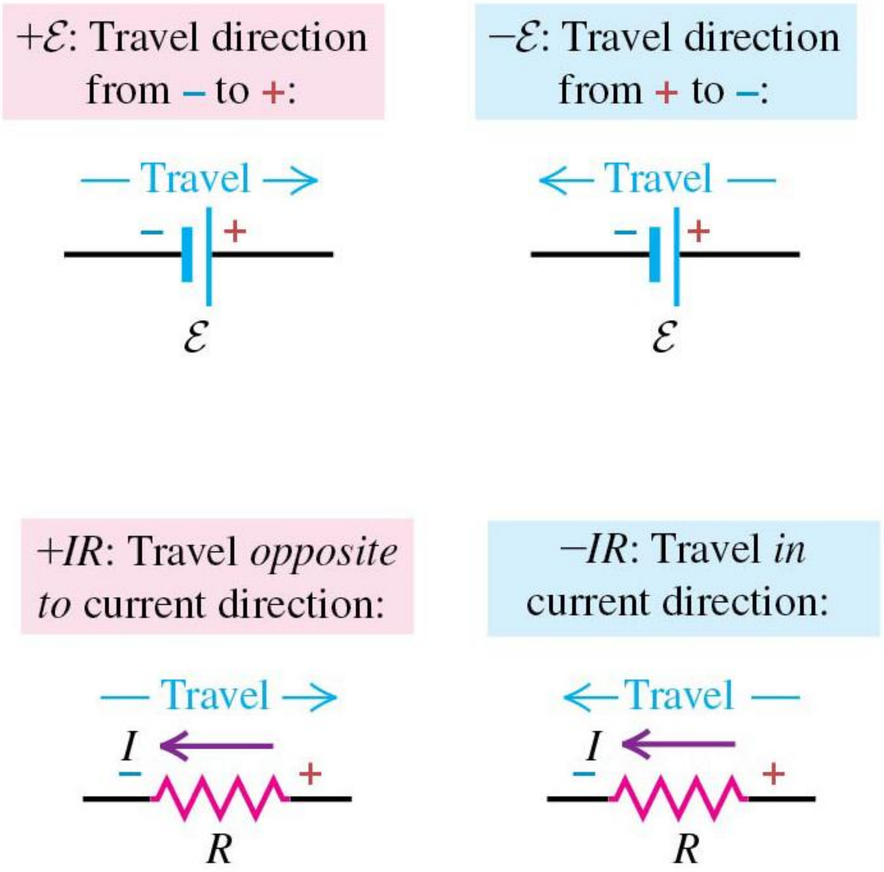
- The sign convention for the loop rule is very important!
(next slide)

Loop rule – sign convention

- For **emf** sources,
  "travel" from – to + → $+\varepsilon$
  "travel" from + to – → $-\varepsilon$

- For **resistors**,
  "travel" against I → $V = +IR$
  "travel with I → $V = -IR$

- "Travel" is the direction we imagine going around the loop with.
  - Not necessarily same direction as I.



*Junction rule equation $\boxed{I_3 = I_1 + I_2}$

Loop rule equations

- For the **left** small loop 1, starting from the top right corner and traveling counterclockwise:

$$\boxed{+\varepsilon_1 - I_1(r_1 + R_1) - I_3R_3 = 0}$$

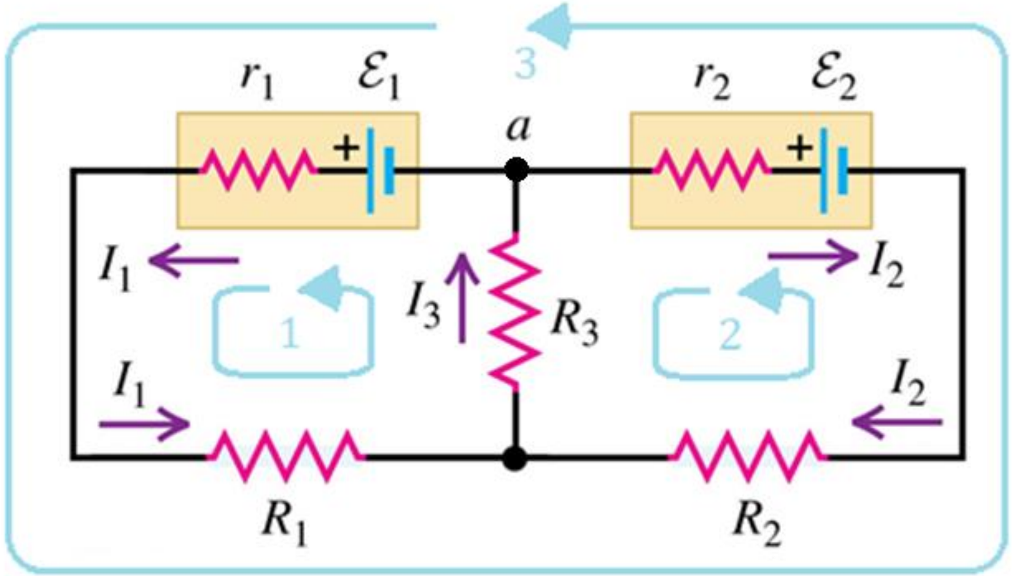
- For the **right** small loop 2, starting from the top right corner and traveling counterclockwise:

$$\boxed{+\varepsilon_2 + I_3R_3 + I_2(r_2 + R_2) = 0}$$

- For the **full** loop 3 starting from top right corner and traveling counterclockwise.

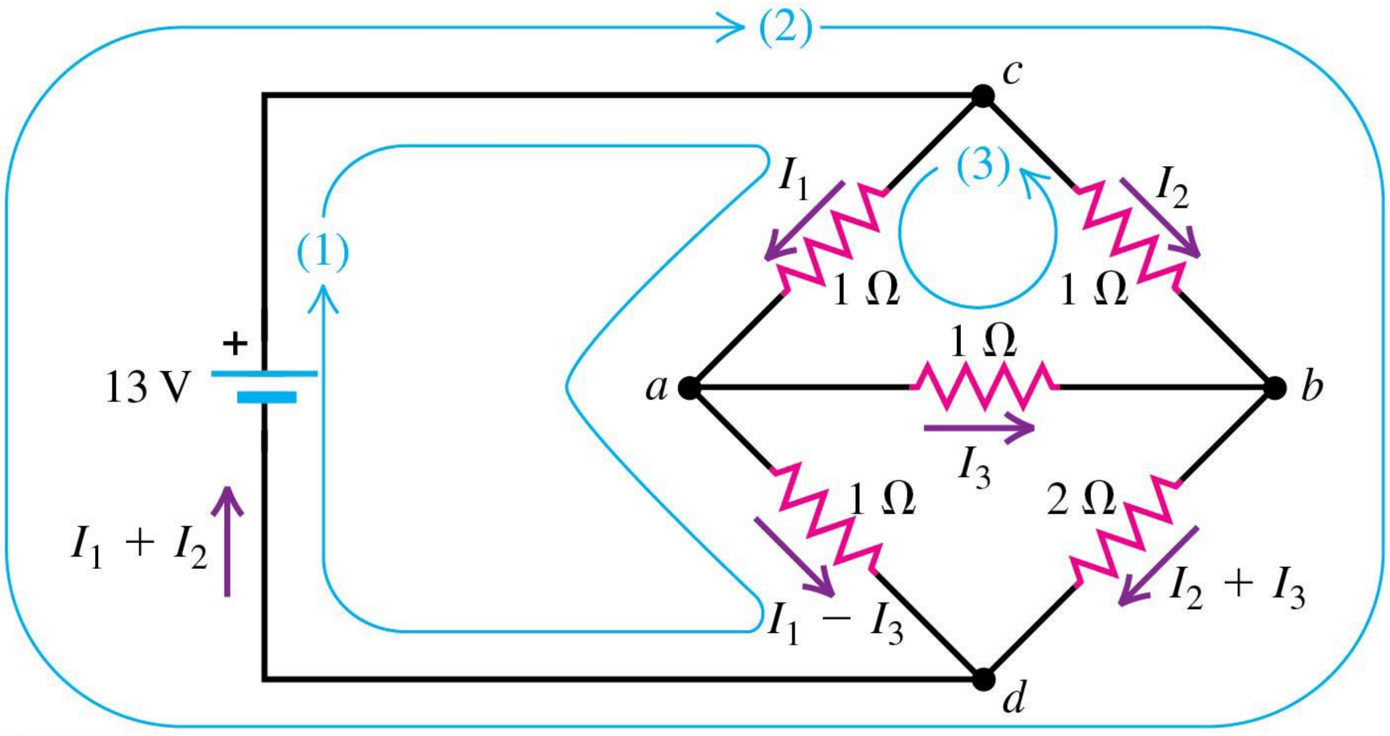
$$\boxed{+\varepsilon_2 + \varepsilon_1 - I_1(r_1 + R_1) + I_2(r_2 + R_2) = 0}$$

- It's possible the directions for the current are not correct! When solving you will get a *negative* value of current if you chose wrong.

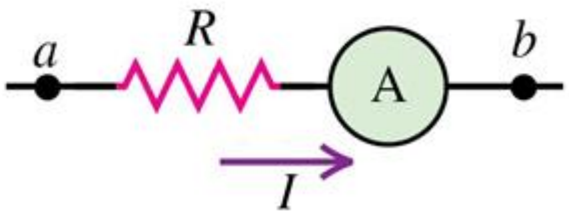


Ex. 26.6 – Complex network

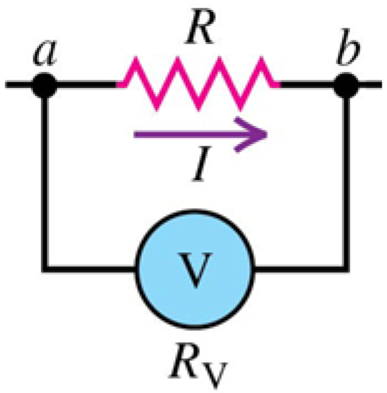
- The figure shows a "bridge" circuit. Use Kirchhoff's rules to find the current in each resistor and the equivalent resistance of the network of five resistors.



Electrical measuring instruments

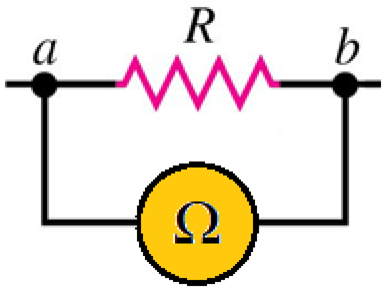


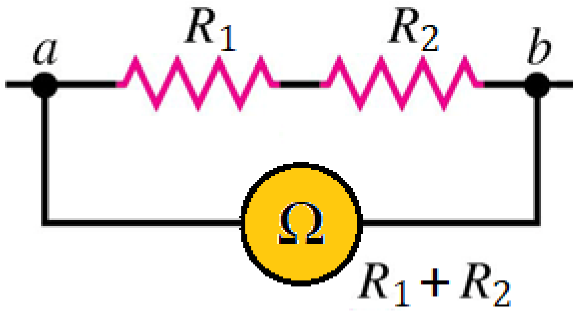
- An **ammeter** measures the current passing through it. Must be connected in series with the circuit elements.



- A **voltmeter** measures the potential difference (voltage) between two points. Must be connected in parallel with circuit elements.

- An **ohmmeter** measures the resistance of a resistor or combination of resistors. Must be connected in parallel.



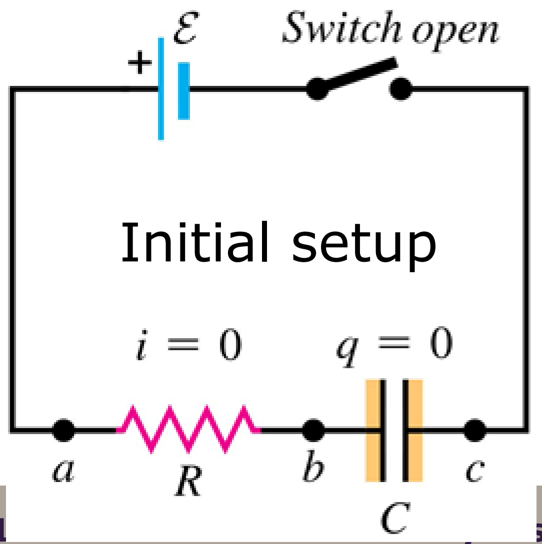


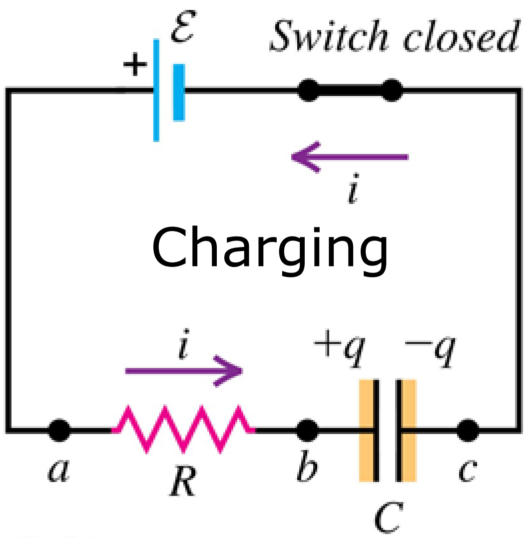
R-C circuits

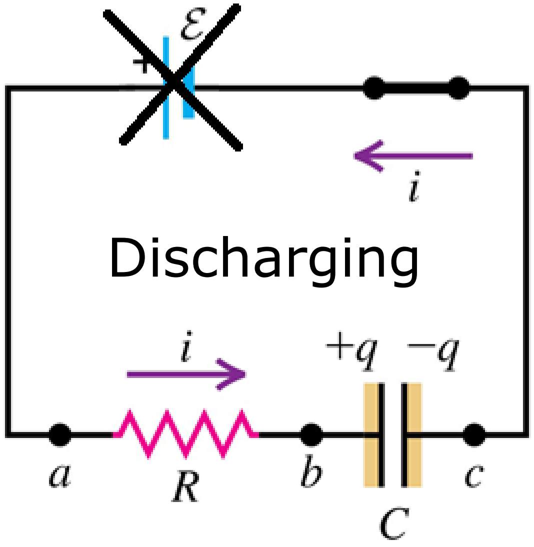
- In a DC circuit with a capacitor, the current *may vary with time.*
  - Magnitude will change (not direction).

- An **RC circuit** combines a capacitor and resistor in series.

- Two situations to consider: **charge** and **discharge**.





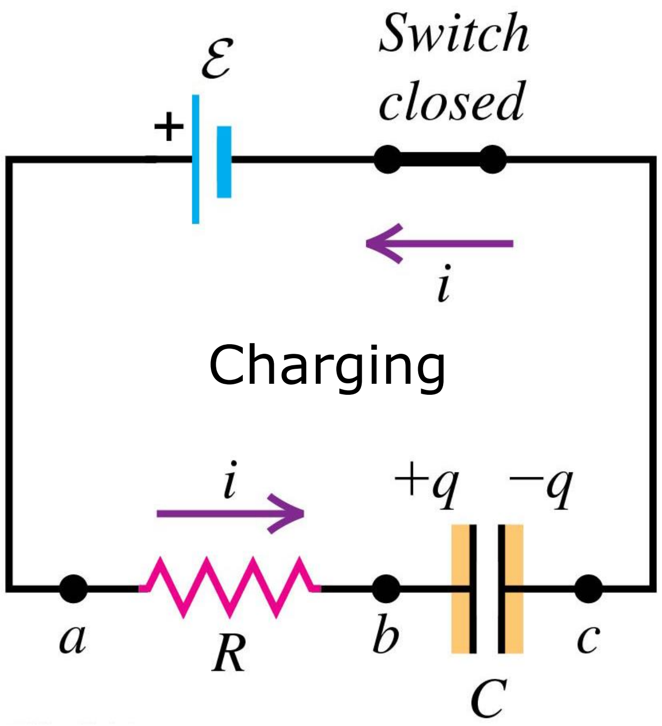


R-C circuits – Charging

- Circuit is complete (at time $t = 0$), **charging** begins.
  - Constant emf, zero r, wires have no R.
- At $t = 0$, $Q_i = 0$ and $\boxed{I_0 = \varepsilon/R}$.
- As t increases, Q increases, i decreases.
- Eventually, charge is max and current is 0.

  $$\boxed{Q_f = C\varepsilon},\ i = 0$$

- How do q and i vary with time?



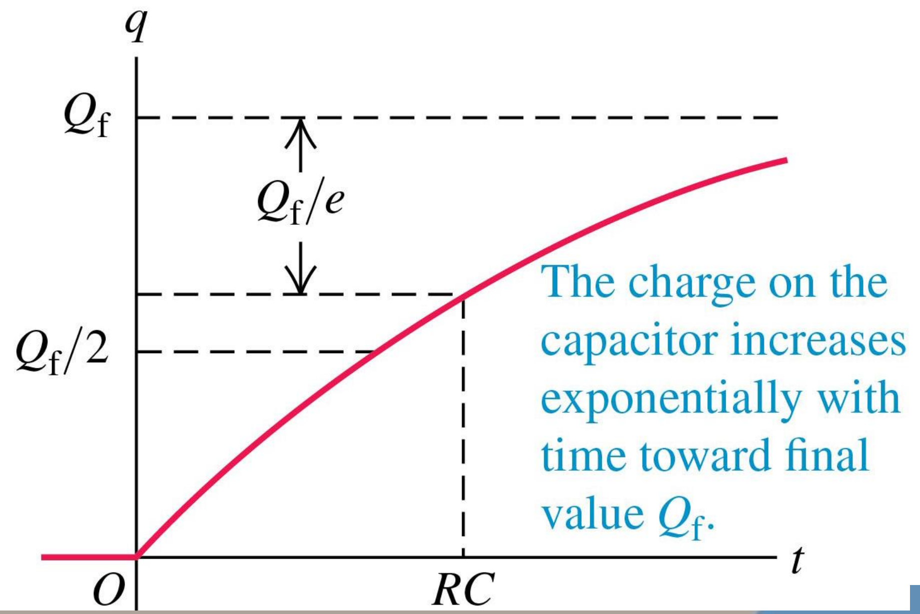
R-C circuits – Charging

- Analysis: start with loop rule, $\varepsilon - iR - \frac{q}{C} = 0$, then solve for i and set $i = dq/dt$. Separate dq and dt then integrate both sides.

$$\boxed{q_c} = C\varepsilon\left(1 - e^{-t/RC}\right) = \boxed{Q_{\mathrm{f}}\left(1 - e^{-t/RC}\right)}$$

- We see that the charge increases exponentially with $(1 - e^{-x})$ relationship.

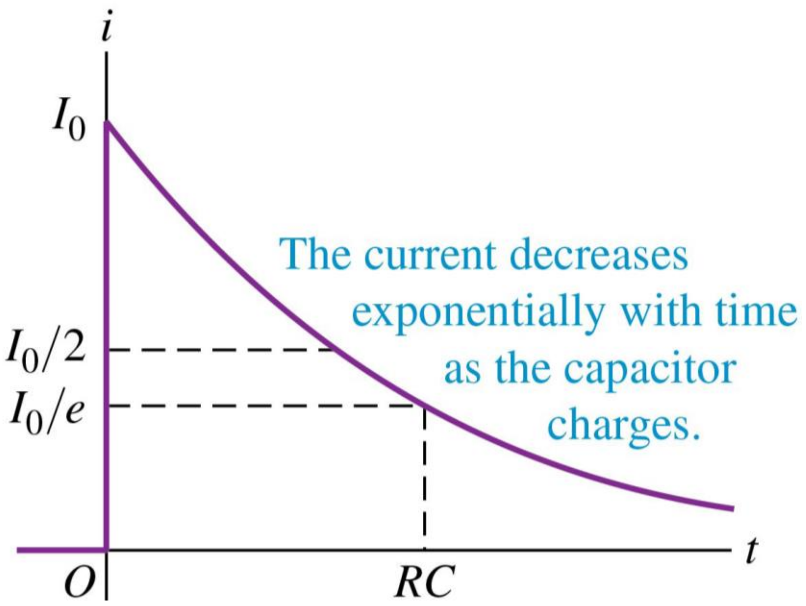
- We call $t = \boxed{\tau = RC}$ the time constant of the circuit.

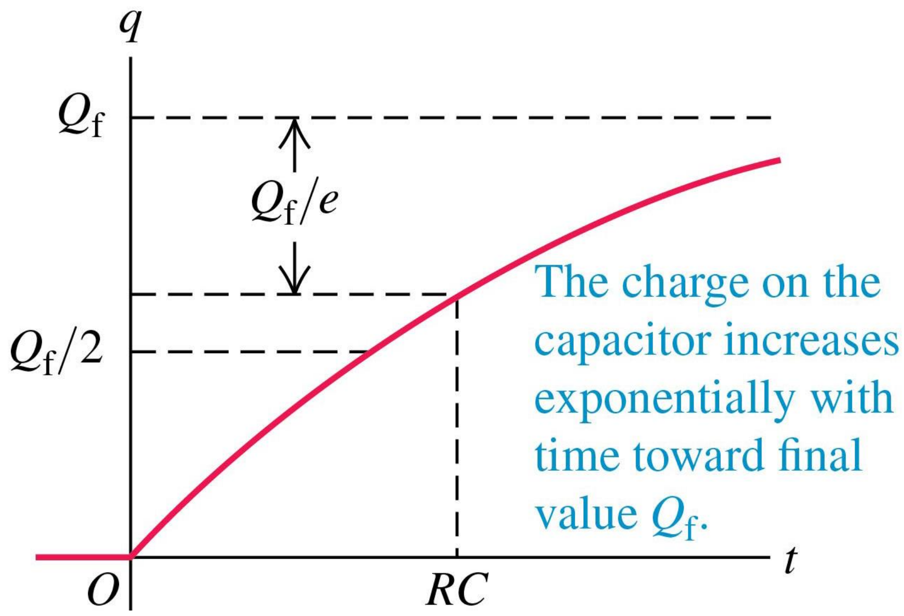


R-C circuits – Charging

- The charge increases exponentially and the current will decrease exponentially:

$$\boxed{i_c} = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/RC} = \boxed{I_0 e^{-t/RC}}$$





R-C circuits – Time constant τ

- When the time is equal to $t = \tau = RC$ then we have:

$$e^{-t/RC} = e^{-RC/RC} = e^{-1} = 0.368 = 36.8\%$$

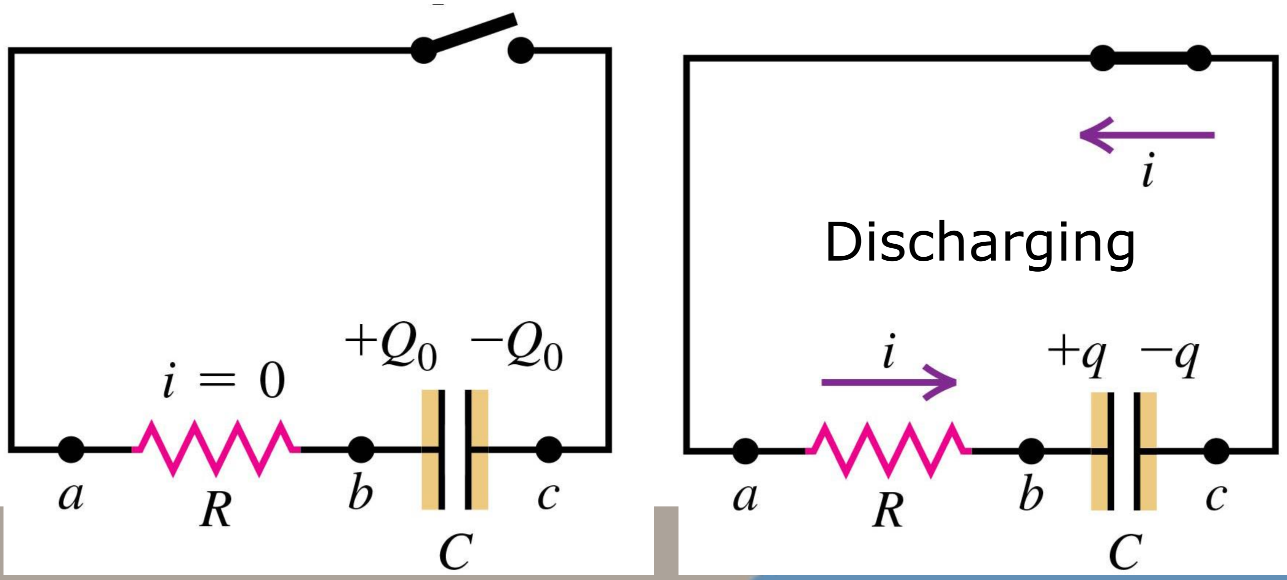
- Therefore after time τ the current has been reduced to 36.8% of it's original value: $i_c = I_0 e^{-1} = \boxed{0.368 I_0}$

- In the same amount of time, the capacitor has charged to 63.2% of it's max charge: $q_c = Q_\mathrm{f}(1 - e^{-1}) = \boxed{0.632 Q_\mathrm{f}}$

Time	q_c	i_c
τ	63.2%	36.8%
2τ	86.5%	13.5%
3τ	95.0%	5.0%
4τ	98.2%	1.8%

R-C circuits – Discharging

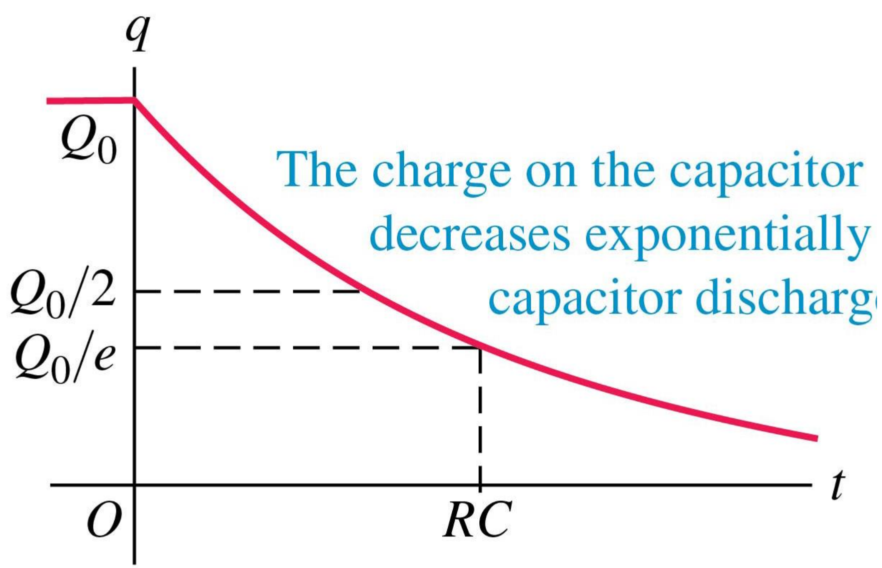
- The figures show a sample for **discharging a capacitor**.
- Before the switch is closed, the capacitor is at charge $\boxed{q = Q_0}$ and the current is $i = 0$.
- When the switch is closed, the capacitor discharges through the resistor. The initial current is $\boxed{I_0 = -Q_0/RC}$.
- Both the charge on the capacitor and the current in the circuit will slowly decrease until they reach 0.

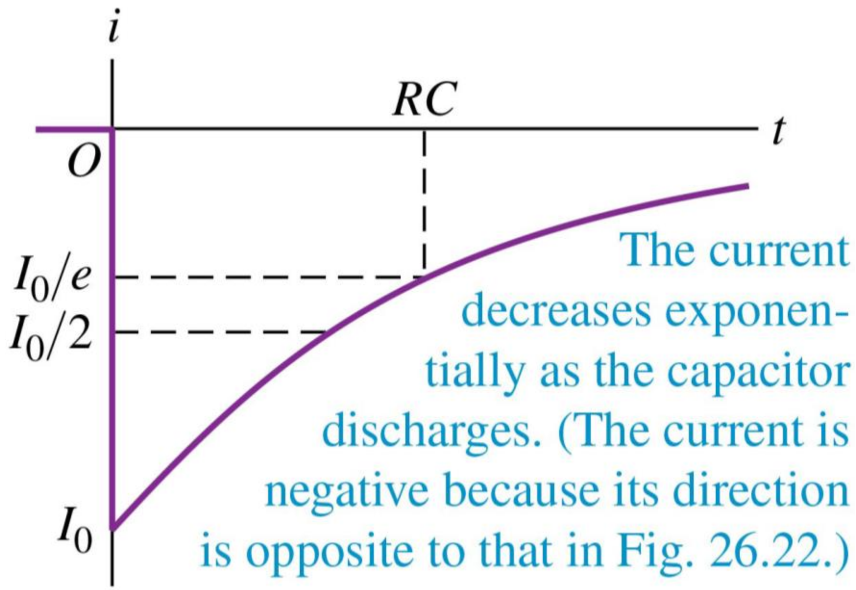


R-C circuits – Discharging

- We perform same analysis as before but with equation $iR - \frac{q}{C} = 0$ then solve for i and set $i = dq/dt$. Separate dq and dt then integrate both sides.

$$\boxed{q_d = Q_0 e^{-t/RC}} \text{ and } \boxed{i_d = I_0 e^{-t/RC}}$$





R-C circuits – Energy and power

- Recall equations for power for emf/resistor/capacitor:

$$P = V_{ab} I = I^2 R = \frac{Q}{C} I$$

- In this case, **power** is rate at which energy is delivered.
For the RC circuit we get:

$$P = \varepsilon i = i^2 R + \frac{iq}{C}$$

- **battery** delivers energy to circuit: $\boxed{\varepsilon i}$
- electrical energy is dissipated in **resistor**: $\boxed{i^2 R}$
- energy is stored in the **capacitor**: $\boxed{iq/C}$