Chapter 12

- Fluid mechanics

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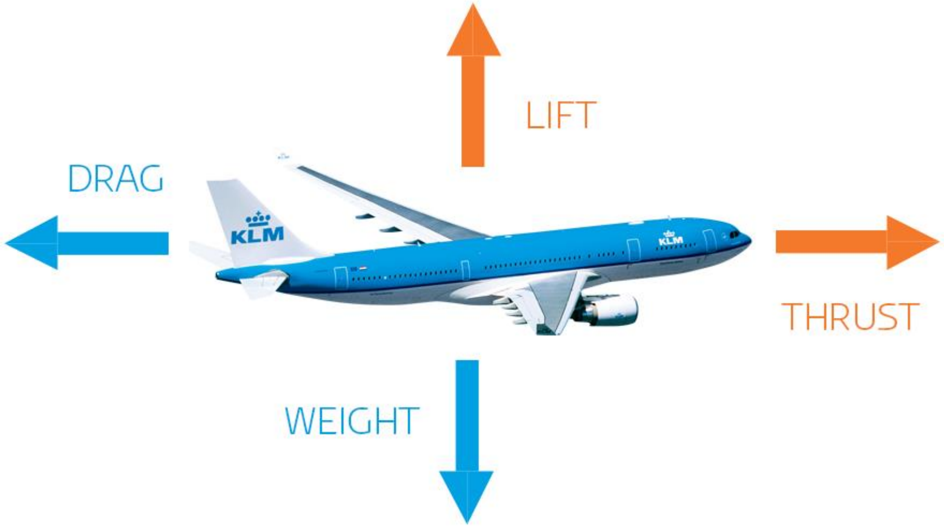
Learning Goals

- Density of a material (density of a body)
- Pressure in a fluid
- Buoyancy force and Archimedes' principle
- Fluid Flow (dynamics…ish)
  - Continuity equation
  - Bernouilli's equation

Introduction

- This chapter will focus on the physics of fluids such as liquids and gases and the forces that act on them.

- We will study concepts such as density, pressure, buoyancy, (statics) and flow (dynamics).



- Example, how does an airplane stay in the air?
  - Engine propels it forward at high speed.
  - Air flows rapidly over the wings which throws air to the ground.
  - This generates an upward force called “lift” that overcomes the plane’s weight.

Gases, liquids, and density

- A **fluid** is any substance that can flow and change the shape of the volume that it occupies (gases and liquids can do this).
  - Liquids have *cohesion*, gases do not.

- The density of a material (fluid or solid) is defined as its mass per volume (*volumetric mass density*):

$$\boxed{\rho = \frac{m}{V}}$$

  typically measured in $\mathrm{kg/m^3}$ (SI units).

- Metals are typically more dense than liquids. ($3 - 10 \times 10^3\ \mathrm{kg/m^3}$ compared to $\sim 1 \times 10^3\ \mathrm{kg/m^3}$)
  - Some solids have low density like styrofoam.

Pressure in a fluid

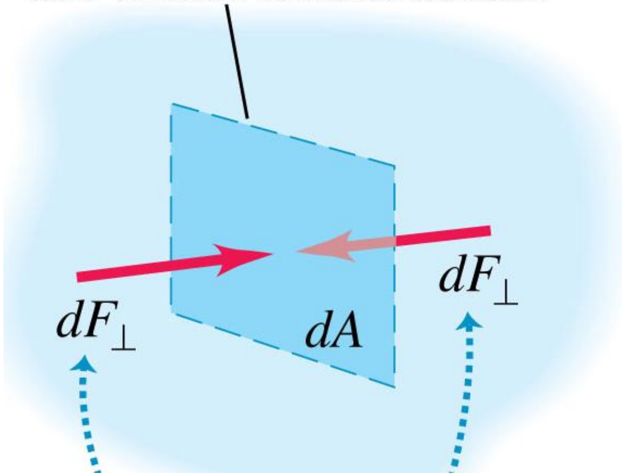
- A fluid exerts a force perpendicular to any surface in contact with it, such as a container wall or a body immersed in the fluid.
- Consider a small surface of area dA within a fluid at rest.
- The pressure p at any point on the surface is:

$$p = \frac{dF_\perp}{dA}$$

with unit pascal ($1\ \mathrm{Pa} = 1\ \mathrm{N/m^2}$).

If the pressure is the same at all points:

$$\boxed{p = \frac{F_\perp}{A}}$$



Pressure and depth

- As you go deeper into fluid, the weight of the fluid can't be ignored when calculating pressure.

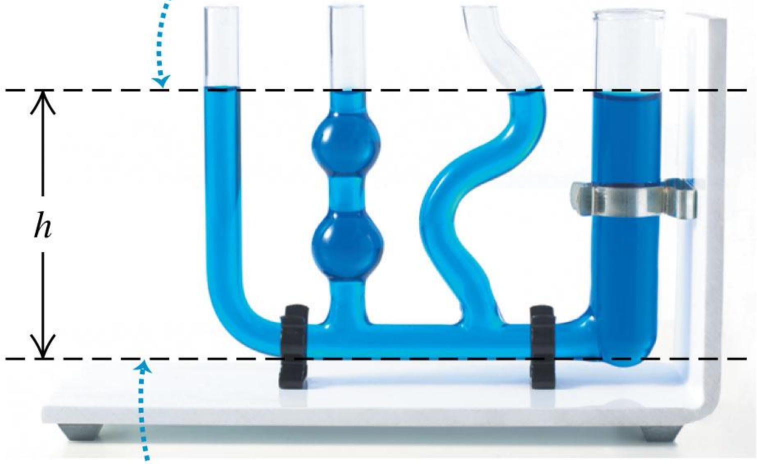
- A simple equation to calculate the pressure at a certain depth in a fluid is:

$$\boxed{p = p_0 + \rho g h}$$

where p_0 is the pressure at the surface.

- This tells us that if we increase the pressure p_0 at the surface then the pressure increases inside the fluid proportionally (**Pascal's law**).

The pressure at the top of each liquid column is atmospheric pressure, p_0.



The pressure at the bottom of each liquid column has the same value p.

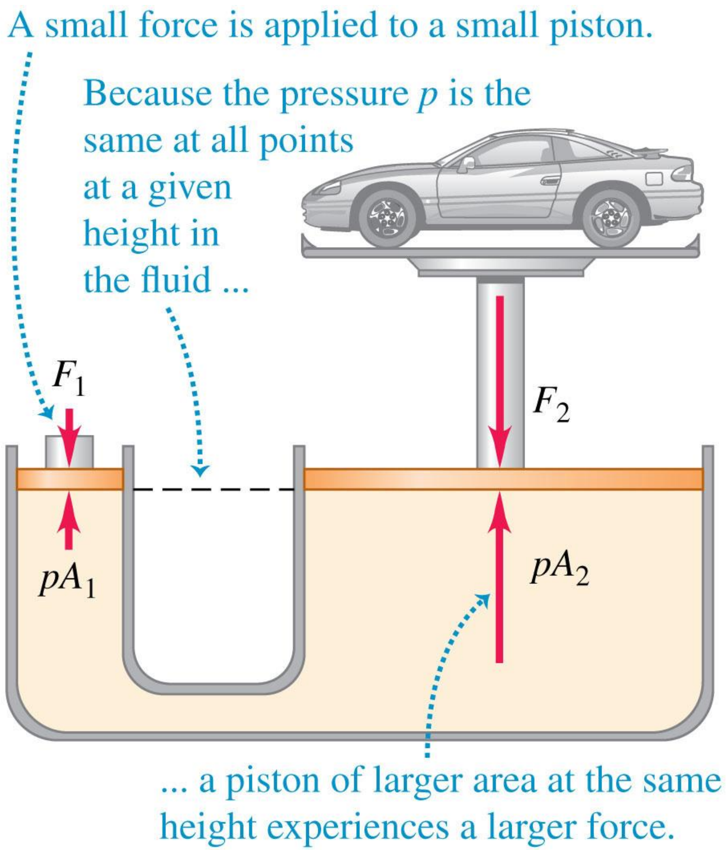
Pascal's law

- **Pascal's law** describes what we just realized:

  Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel.

- Consider the hydraulic lift.
  - A piston exerts a force F_1 on a small area A_1. The applied pressure $p = F_1/A_1$ is transmitted to the other piston with area A_2.

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \rightarrow \quad \boxed{F_2 = \frac{A_2}{A_1} F_1}$$

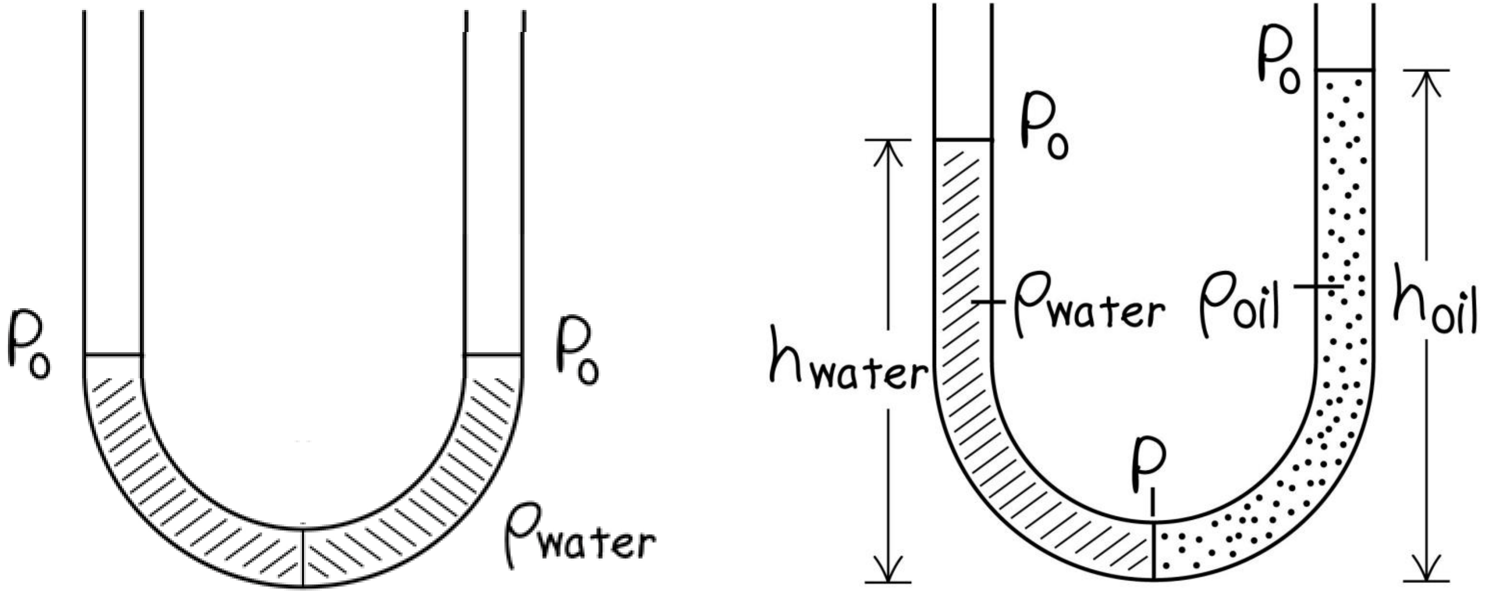


Ex. 12.4 – A tale of two fluids

- A manometer tube is partially filled with water. Oil (which does not mix with water) is poured into the right arm of the tube until the oil-water interface is at the midpoint (center) of the tube as shown in the figure. Both arms of the tube are open to air. Find a relationship between the heights h_{oil} and h_{water}.

  Ie. Which height will be higher?

  Ie. The drawing shows $h_{oil} > h_w$ is this correct?



- Note: $\rho_{water} = 1000 \frac{\text{kg}}{\text{m}^3}$

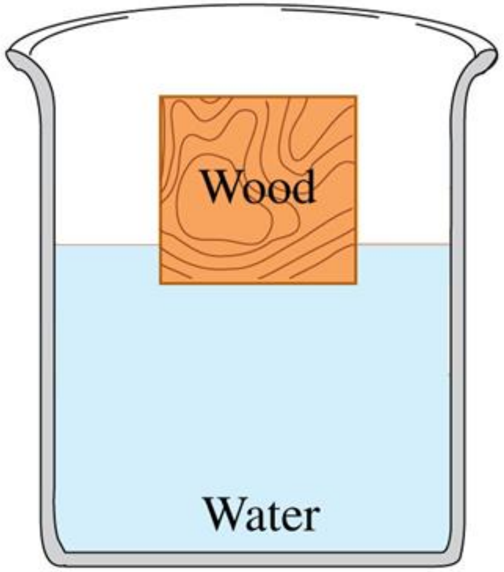
  $\rho_{oil} = 850 \frac{\text{kg}}{\text{m}^3}$

Buoyancy

- A body immersed in water seems to weigh less than when it is in air. If a body is less dense than the fluid, it floats.
  (Eg. human body in water, helium balloon in air)

- These are examples of **buoyancy** as described by **Archimedes's Principle**:

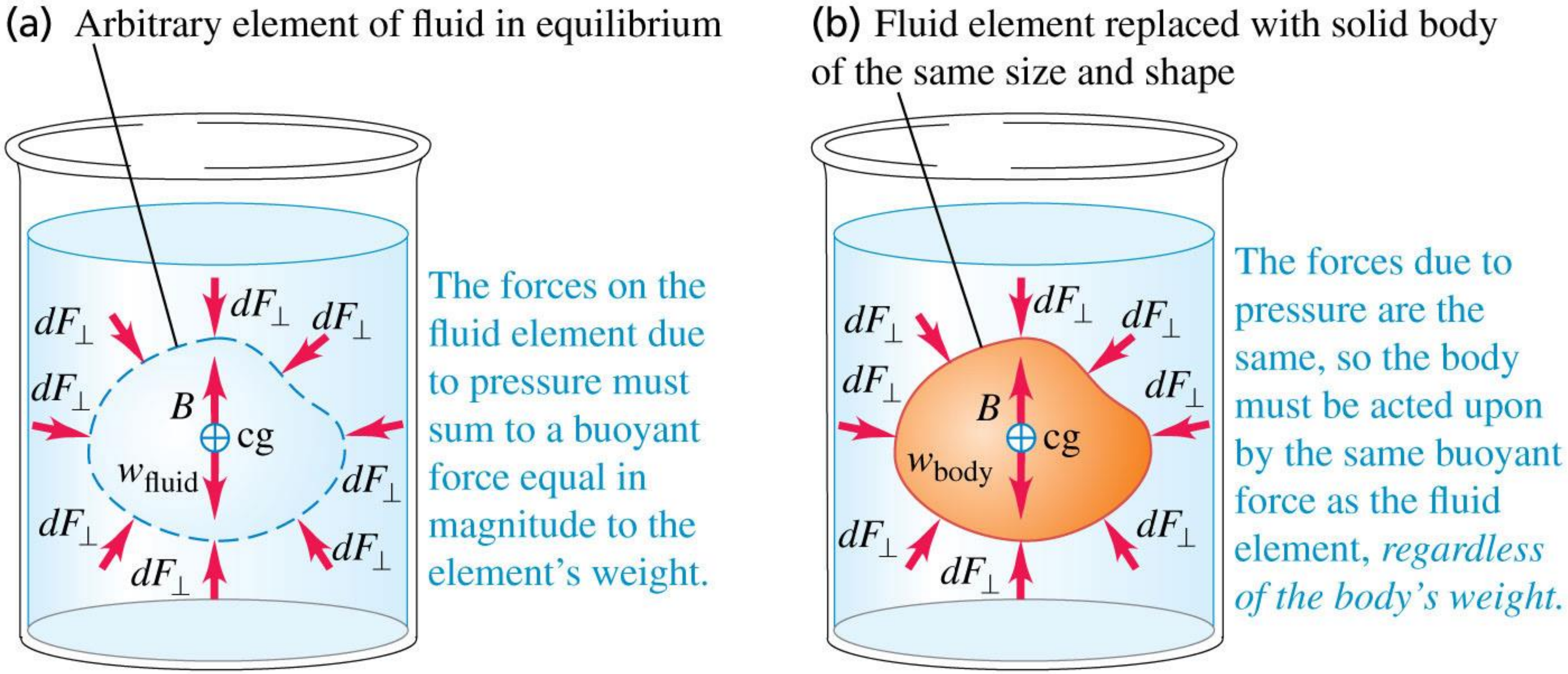
  When a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.



- The upward force is called the **buoyant force** on the body.

Diagram of Archimedes's principle

- For both an arbitrary element of fluid or a solid body with the same size/shape in equilibrium, an upward force must balance the downward gravitational force.

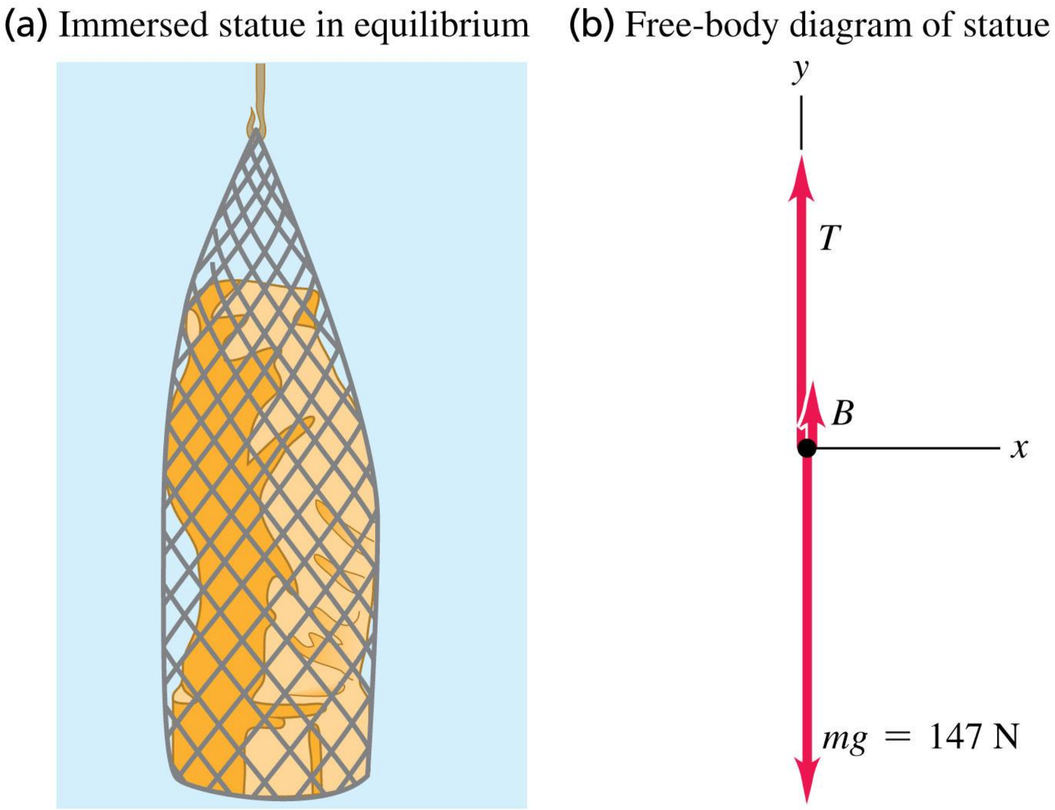


Ex. 12.5 – Buoyancy Force

- A 15.0 kg solid gold statue is raised from the sea bottom. What is the tension in the hoisting cable when the statue is
  (a) at rest and complete underwater and
  (b) at rest and completely out of the water?

- This problem shows us how to calculate the buoyancy force:
  $$\boxed{B = \rho V g}$$

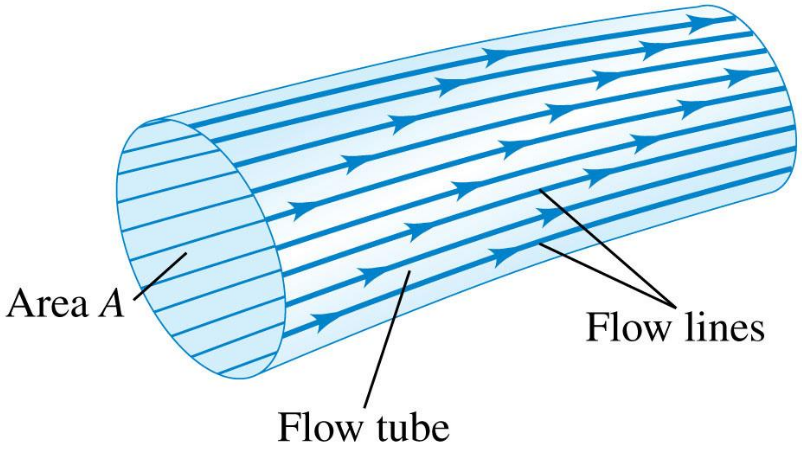
- Notes:
  $\rho_{gold} = 19.3 \times 10^3 \ \mathrm{kg/m^3}$
  $\rho_{sw} = 1.03 \times 10^3 \ \mathrm{kg/m^3}$
  $\rho_{air} = 1.2 \ \mathrm{kg/m^3}$



Fluid flow

- The path of an individual particle in a moving fluid is called a **flow line**.
- During **steady flow**, the overall flow pattern doesn't change so every element that passes through a given point follows the same flow line.
- During steady flow, no fluid will cross the side walls of a given **flow tube**.
- **Laminar flow** describes a smooth flow.

  Turbulent flow describes a chaotic or erratic pattern of flow.



The continuity equation

- Consider the figure where we have a flow tube that changes cross-sectional area.

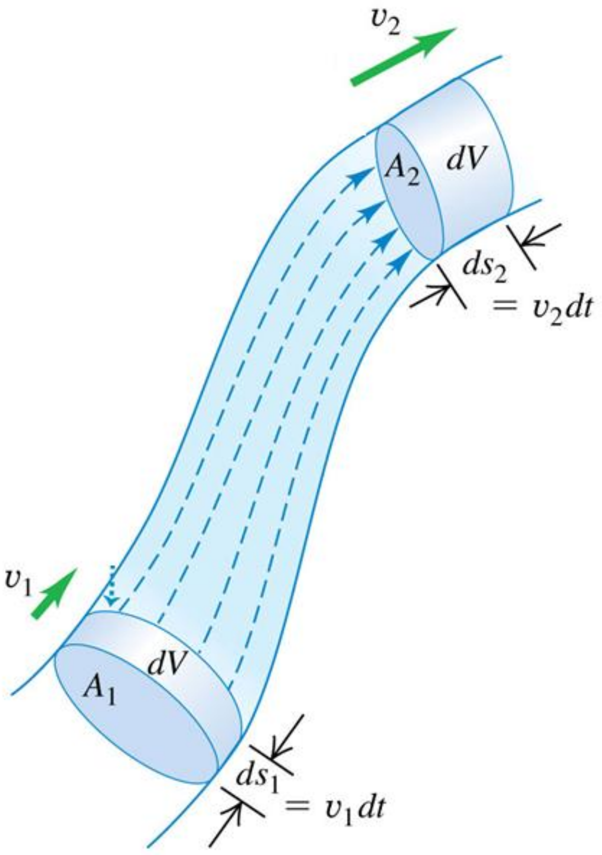
- At area A_1, the speed of fluid flow is v_1 and similarly we have speed v_2 for area A_2.

- Since the mass of the fluid does not change during flow, we can express the areas and speeds as:

$$\boxed{A_1 v_1 = A_2 v_2}$$

where Av is called the **volume flow rate** $(= dV/dt)$.

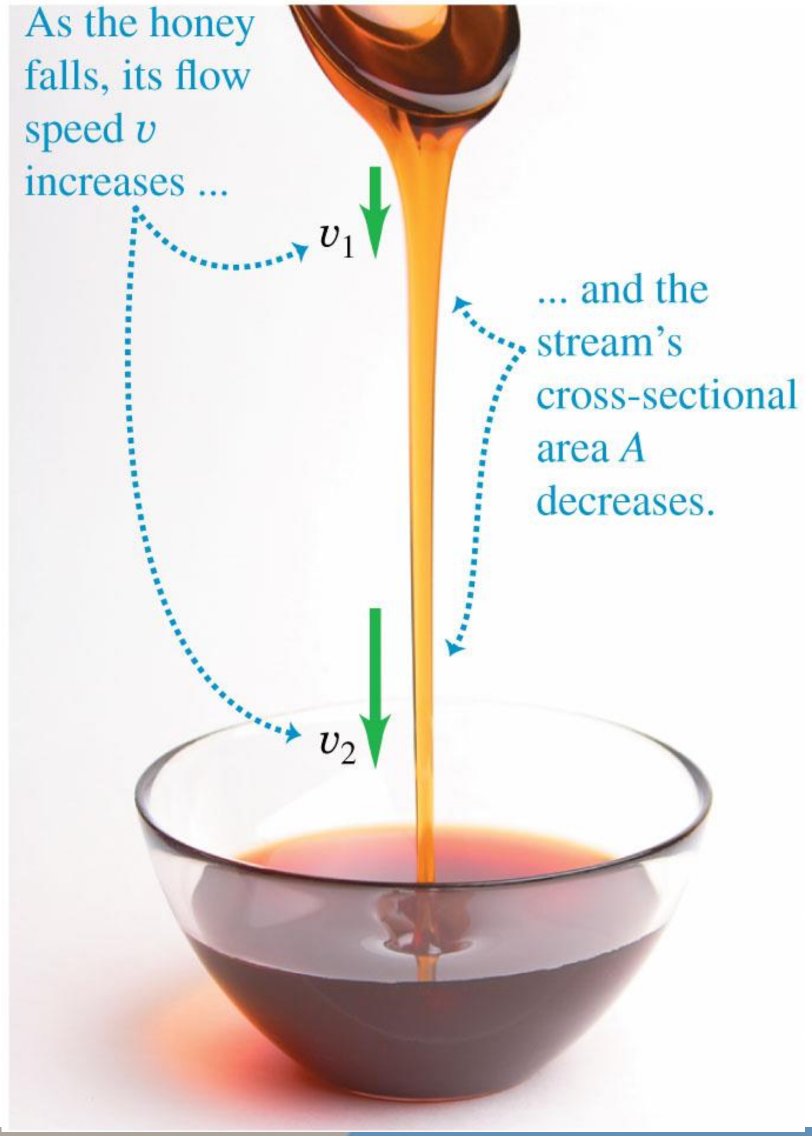
- It's simple to see that if we decrease the cross-sectional area, we can increase the speed of flow (think of a syringe).



The continuity equation in action

- The syringe is one demonstration of the continuity equation.

- Another is the stream of a thick liquid like honey or molasses.
- Near the spoon, the velocity of the honey is low but as it falls, the velocity increases.
- Accordingly, the cross-sectional area of the honey stream will decrease.



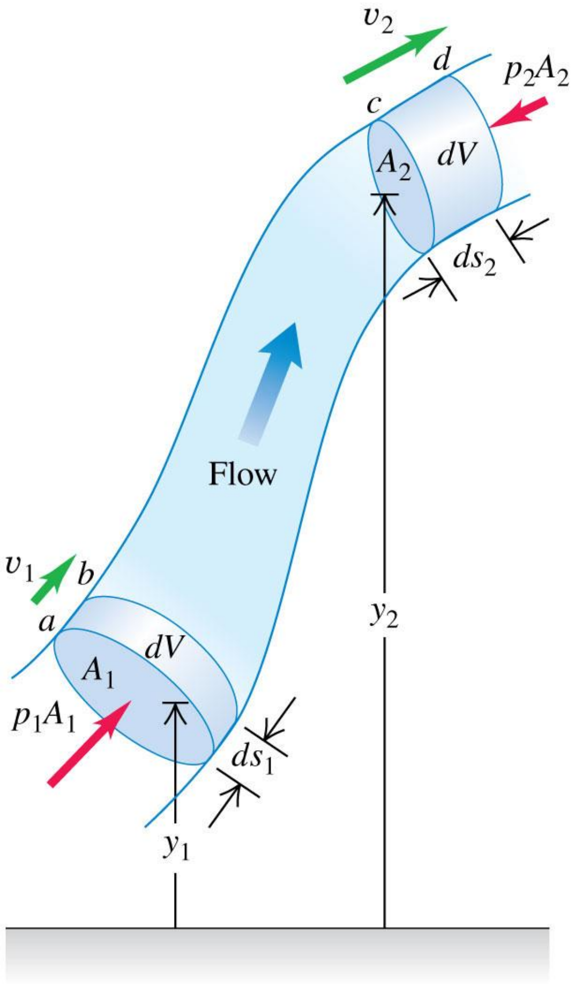
Bernouilli's equation

- In order to analyze more complicated fluid flow (important for most engineers), we need to relate the density, pressure, flow speed, and height of flow. The relation of these properties is **Bernouilli's principle**.

- **Bernouilli's equation** describes all these properties.
  - Derived from the work-energy theorem applied to a fluid.

- The equation is: $\boxed{p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}}$

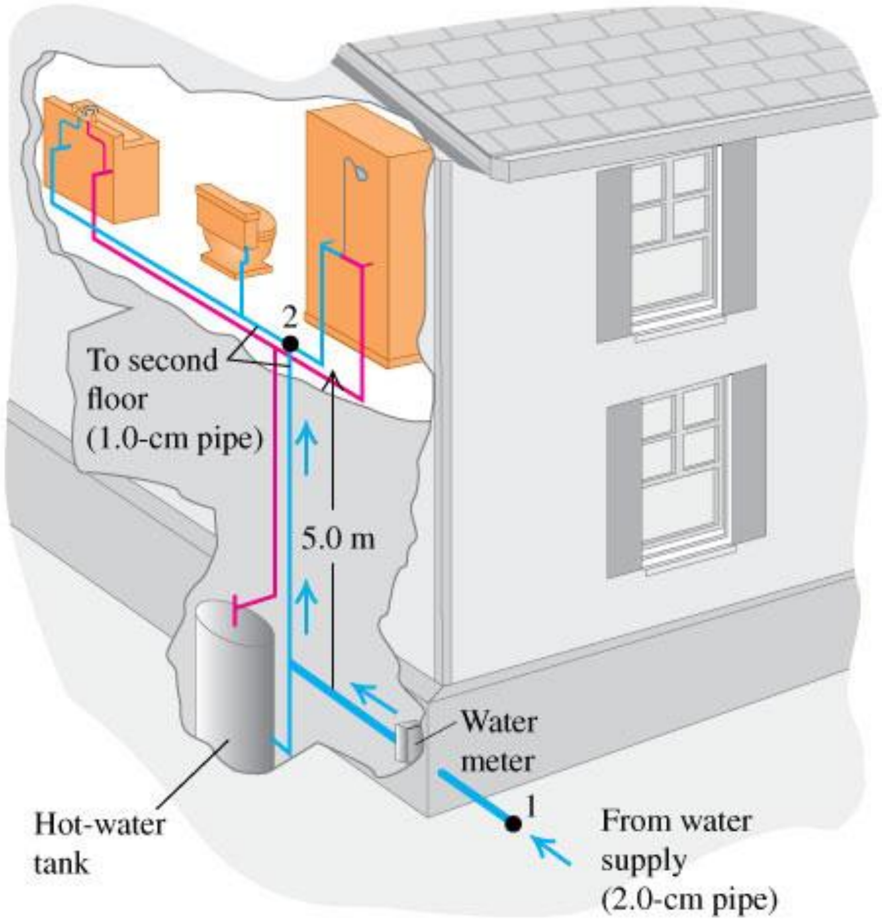
  for an ideal, incompressible fluid.



Ex. 12.7 – Water pressure at home

- Water enters a house through a pipe with ID = 2.0 cm and pressure of 4.0 x 10^5 Pa. An ID = 1.0 cm pipe leads to second floor bathroom 5.0 m above.

- When the flow speed at the inlet pipe is 1.5 m/s, what is:
(a) **flow speed**,
(b) **pressure**,
(c) and **volume flow rate**
in bathroom?



Example: Lift on an airplane wing

- Bernoulli's principle helps to explain how airplanes fly.