Chapter 25

- Current, Resistance, and Electromotive Force

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Learning goals

- Meaning of **electric current** and how charges move in a conductor.
- Calculating **resistance.**
- How **electromotive force (emf)** makes current flow.
- **Energy** and **power** in electrical circuits.

Electric circuits

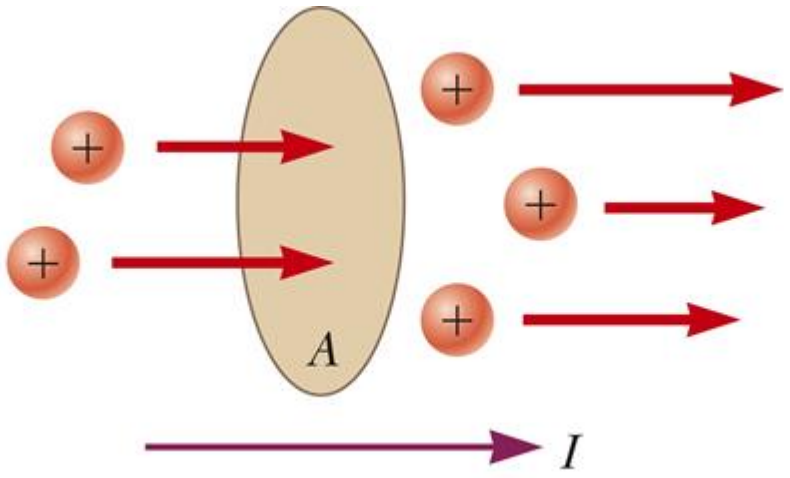
- **Electric circuits** are used in every electronic device (TV, computer, calculator, etc…).
- In a closed circuit, charges can flow through conducting wires (typically electrons).
- **Resistors** are used to control the rate of charge flow.
  - 2nd circuit element we see (1st was capacitor).
- A **battery** can transfer charge and energy to a "load" (like a lightbulb).
  - The current flowing into the load is equal to current flowing out of it.

Electric current

- An **electric current** is the flow of charge from one region to another (think of current in a stream). The charged particles that contribute to current are called **charge carriers**.

- The current (scalar) is defined to be the amount of charge dQ that passes through an area A in time dt:

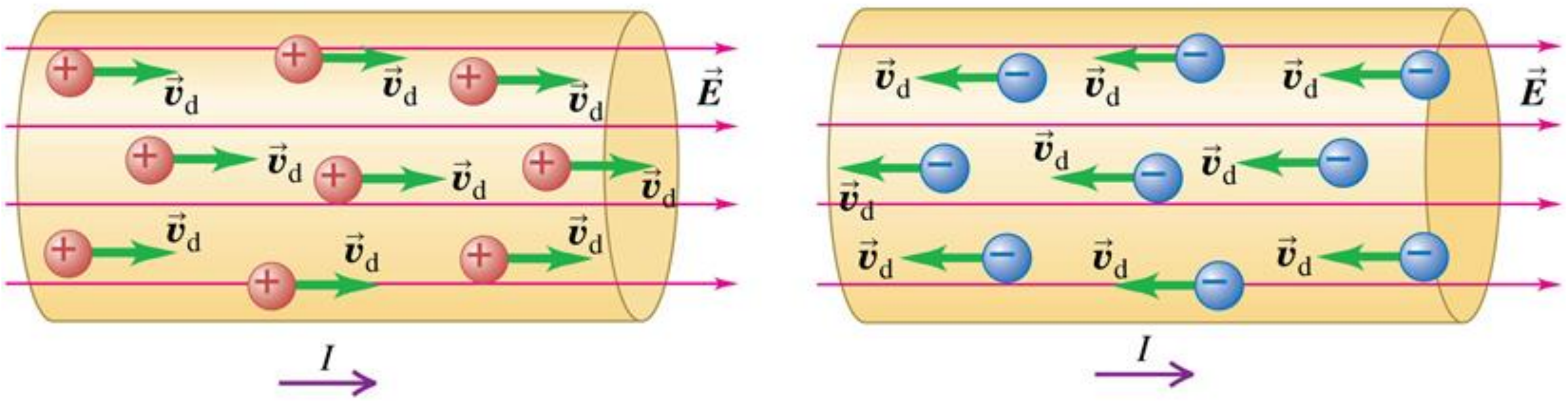
$$\boxed{I = \frac{dQ}{dt}}$$



- The SI unit of current is the ampere: 1 A = 1 C/s

Direction of current

- Current can be produced by the flow of positive, negative, or a combination of both charges.
- The convention we use: current is in direction of $+q's$.
- In a normal electric circuit, charge carriers (electrons) move opposite the direction of current.



Drift speed of charge carriers

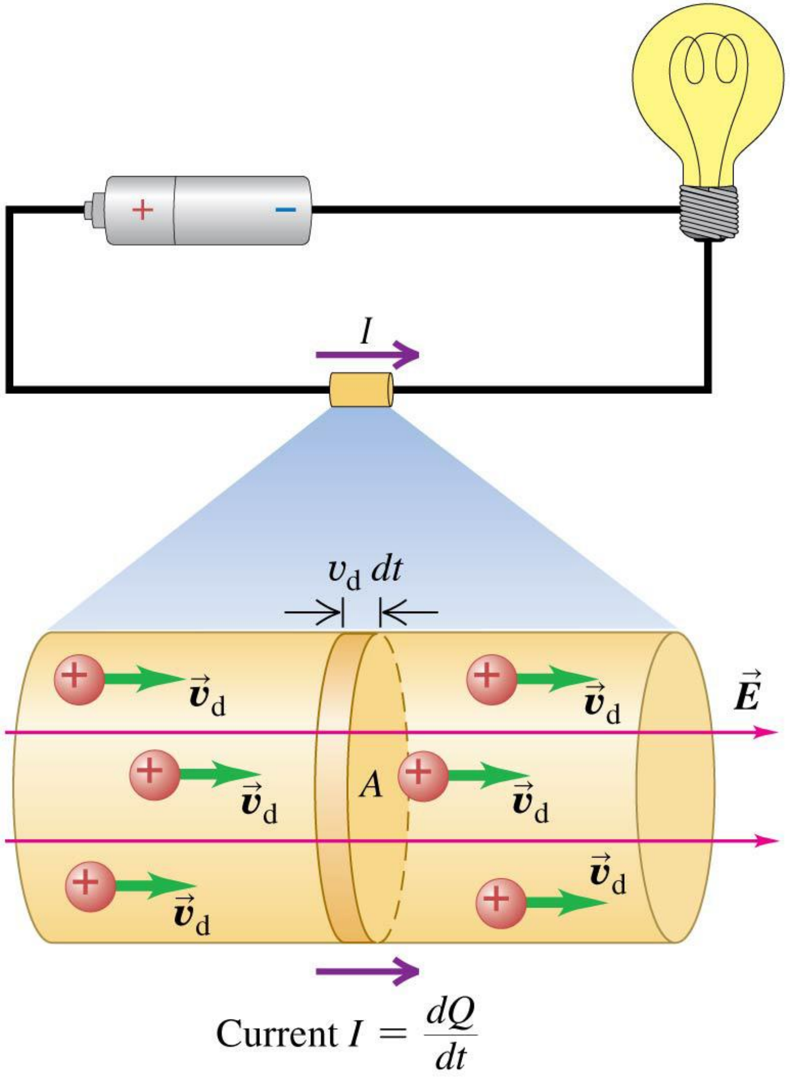
- In a thin slice of wire with area A, thickness $v_d dt$, there are n carriers per volume. The total charge in this area is:

$$dQ = q(nAv_d dt)$$

- Therefore the current, in terms of geometric components, is:

$$I = \frac{dQ}{dt} = nqAv_d$$

$$v_d = \frac{I}{nqA}$$

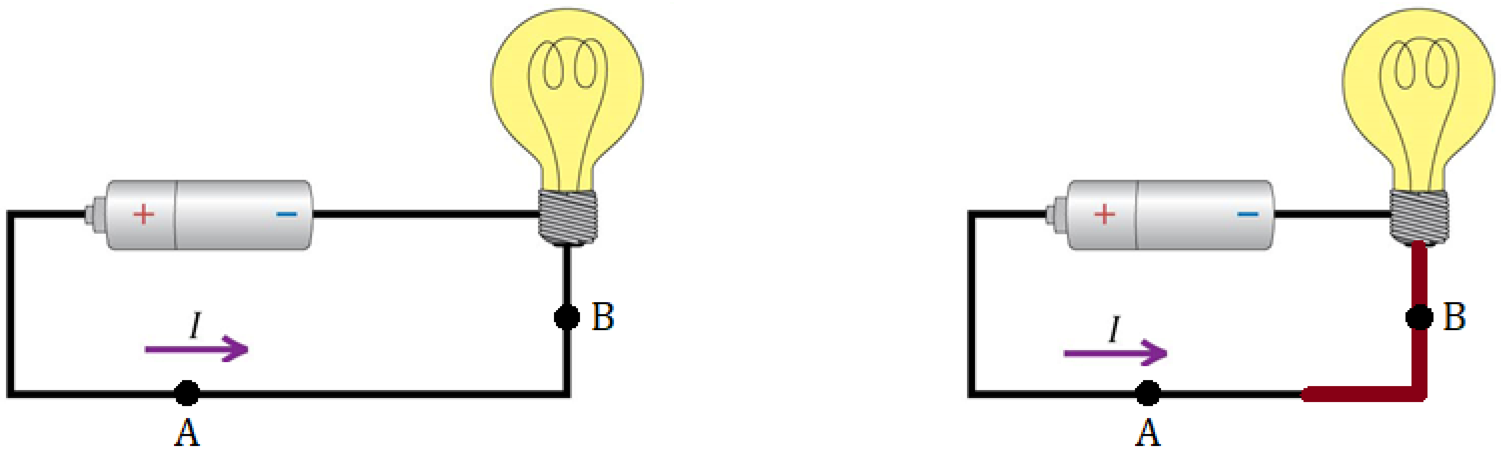


Current density

- The **current density** (vector) is defined as the current per unit area and includes drift velocity:

$$\boxed{\vec{J}} = \frac{I}{A} = \frac{nqA\vec{\boldsymbol{v}}_d}{A} = \boxed{nq\vec{\boldsymbol{v}}_d}$$

- Current density is always in the same direction as the electric field.
  - It describes how charges flow at a given point.



Ex. 25.1 - Current density and drift velocity in a wire

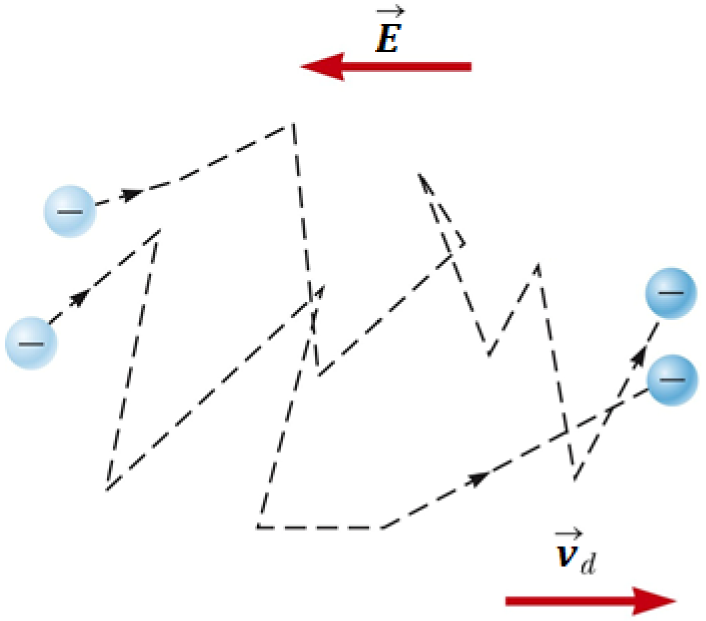
- An 18-gauge copper wire has diameter of 1.02 mm and carries a constant current of 1.67 A to a 200 W lamp. The free electron density in the wire is $8.5 \times 10^{28}\ 1/\mathrm{m}^3$.

  (a) Find the magnitude of the current density.
  (b) Find the drift speed.

$$A = \pi \left(\frac{d}{2}\right)^2 = 8.17 \times 10^{-7}\ \mathrm{m}^2$$

$$\boxed{J} = \frac{I}{A} = \boxed{2.04 \times 10^6\ \frac{\mathrm{A}}{\mathrm{m}^2}}$$

$$\boxed{v_d} = \frac{J}{nq} = \boxed{1.5 \times 10^{-4}\ \mathrm{m/s}}\ \text{(about 0.15 mm/s)}$$



Resistivity

- Current density in a conductor depends on electric field and on material properties.

- For some materials, $\vec{J} \propto \vec{E}$. These materials are said to be "ohmic" and follow **Ohm's law**:

$$\boxed{J = \sigma E} \quad \text{or} \quad \boxed{E = \rho J}$$

  where σ is the **conductivity** and ρ is the **resistivity**.

- A perfect conductor has $\rho = 0$ and perfect insulator: $\rho = \infty$.

- The SI unit for resistivity is $\Omega \cdot \mathrm{m}$.

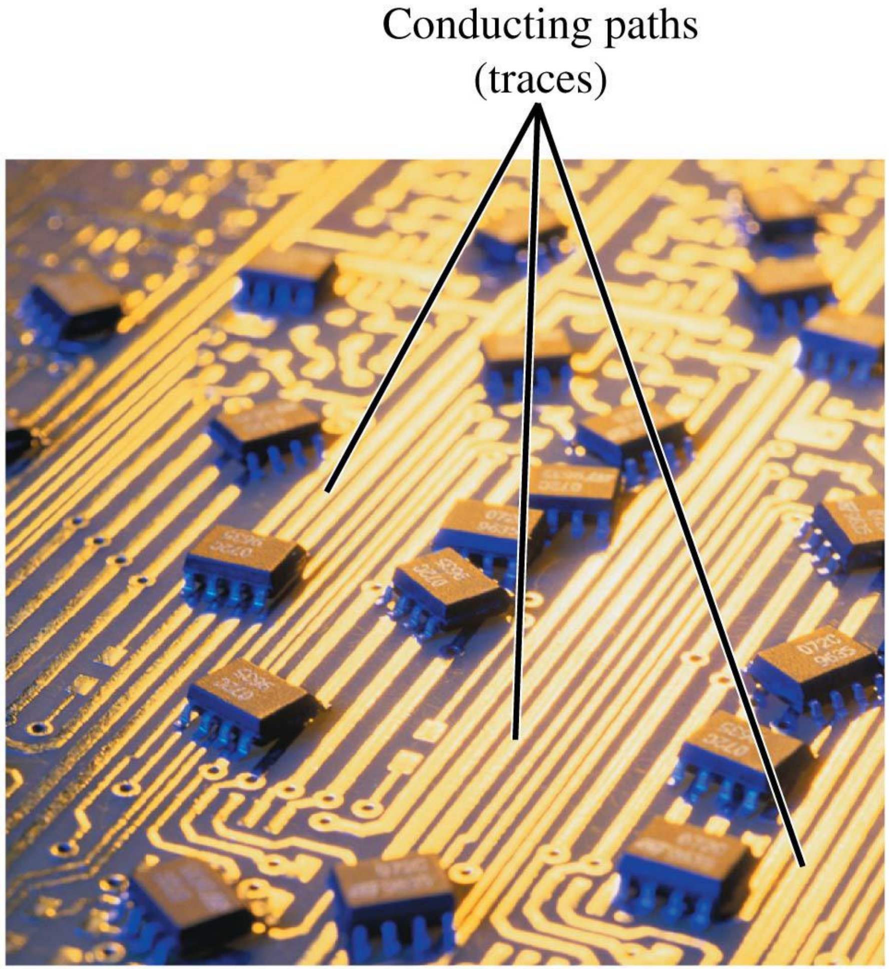
Some resistivities at $T = 20℃$

	Substance	ρ ($\Omega \cdot$ m)
Conductors	Copper	1.72×10^{-8}
	Gold	2.44×10^{-8}
	Lead	22×10^{-8}
Semiconductor:	Pure carbon (graphite)	3.5×10^{-5}
Insulators	Glass	$10^{10} – 10^{14}$
	Teflon	$>10^{13}$
	Wood	$10^{8} – 10^{11}$

Circuits boards and resistivity

- Copper "wires" or traces can be printed directly onto a circuit board's surface.

- The traces can be extremely close together but the board has high resistivity so current can't flow from trace to trace.



Resistivity and temperature

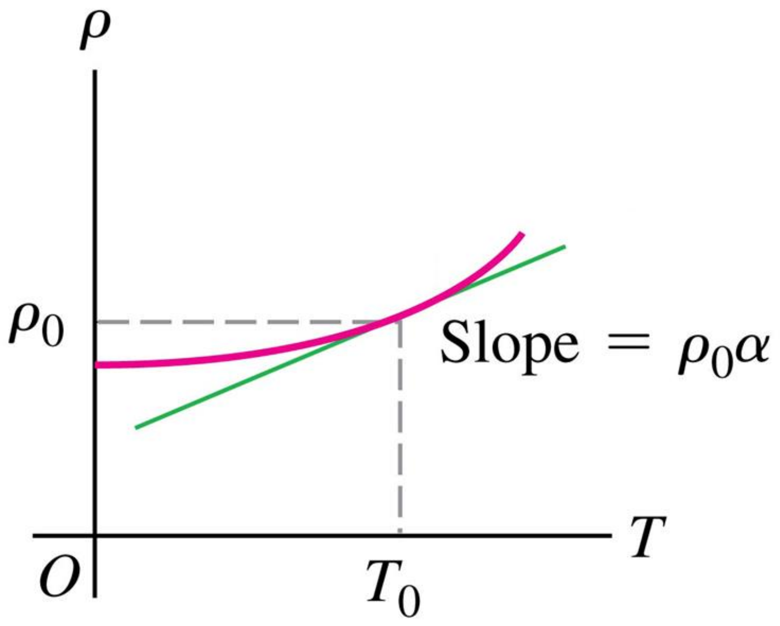
- Resistivity of a metallic conductor (almost) always increases with temperature.

- At higher T, ions in a conductor vibrate more so chance of electron collision with an ion is higher.

- The temperature dependence equation for resistivity is:

  $$\boxed{\rho(T) = \rho_0[1 + \alpha(T - T_0)]}$$

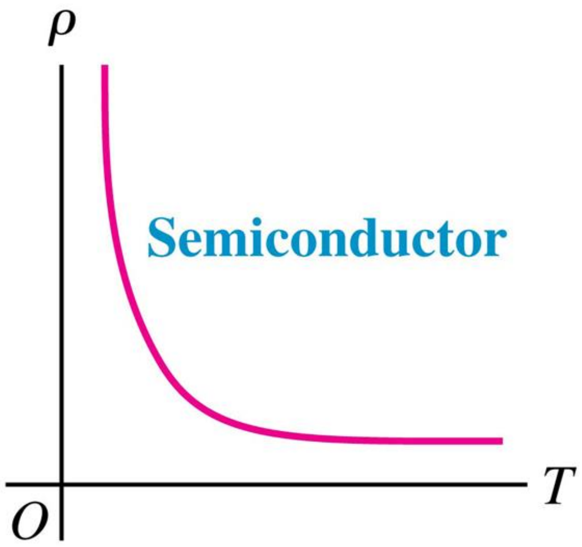
  where α is the temperature coefficient.

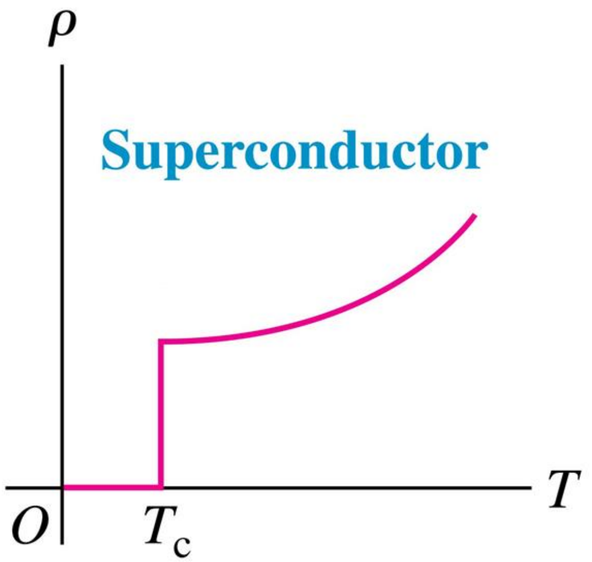


Resistivity and temperature

- For some materials, resistivity decreases for higher T (such as graphite, a semiconductor where α is negative).
  - Electrons "shake loose" when atoms are excited.

- Some materials display a phenomenon called "superconductivity" where below a critical temperature, T_c, the resistivity drops to virtually 0.
  - For Hg, $T_c = 4.15\ \mathrm{K}$, and $\rho_{sc} = 10^{-28}\ \Omega \cdot \mathrm{m}$.





Ohm's law and resistance

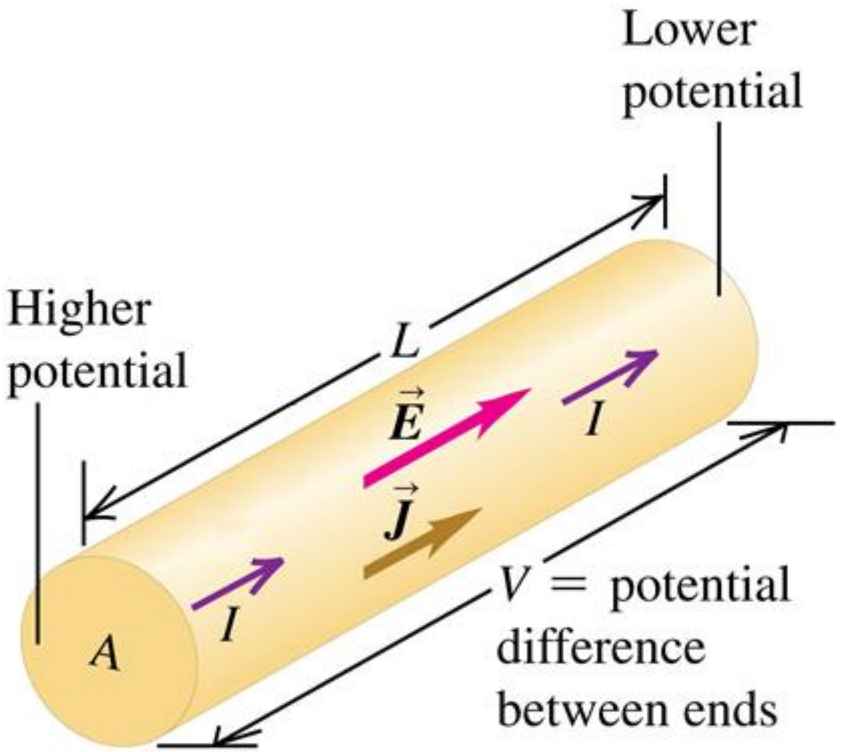
- Recall: Ohm's law $\vec{E} = \rho\vec{J}$.
  - Electric field and current density are hard to measure.
  - We might be more interested in V and I.

- For a conductor with length L, resistivity ρ, and area A.

$$J = \frac{I}{A} \quad \text{and} \quad E = \frac{V}{L}$$

- Put these into equation above to get:

$$\boxed{V = \frac{\rho L}{A} I}$$



Ohm's law and resistance

- Our Ohm's law relation has now been reduced to:

$$V = \frac{\rho L}{A} I$$

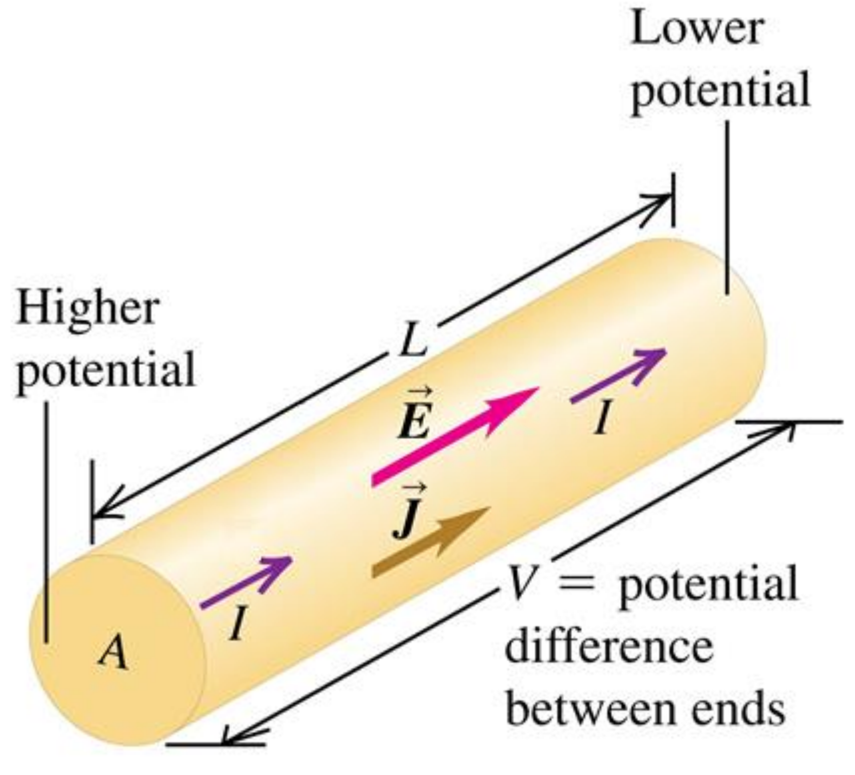
where the proportionality constant between V and I is called the **resistance**:

$\boxed{R = \rho L / A}$ with unit Ohm (Ω).

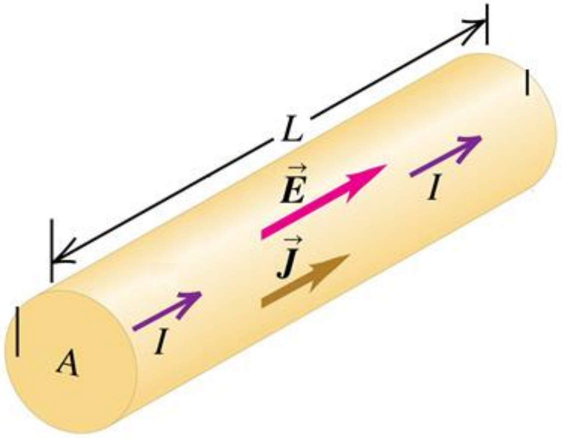
- This gives us a **new form for Ohm's law** which you might have seen before:

$$\boxed{V = RI}$$

which is much easier to verify in a lab setting.



Properties of resistance



- Resistance $R = V/I$ and resistivity ρ are similar but different.
  - Resistance is for specific unit of a conductor
  - Resistivity is inherent to type of conductor (or metal).

- Resistance also increases with temperature for a large range:

$$R(T) = R_0[1 + \alpha(T - T_0)]$$

  - We can exploit this to make very sensitive temperature probes (**resistance thermometers**).

- The SI unit of resistance: 1 **ohm** (Ω) = 1 V/A .

- A **resistor** is a circuit device that has a specific resistance R.

Ex. 25.2 – Field, potential, and R

- The 18-gauge copper wire with diameter 1.02 mm carries current of 1.67 A.

(a) Find the magnitude of the electric field in the wire.

(b) find the potential difference between two points that are 50.0 m apart in the wire.

(c) Find the resistance of a 50.0 m length of this wire.

NB: $\rho_{\mathrm{Cu}} = 1.72 \times 10^{-8}\ \Omega \cdot \mathrm{m}$

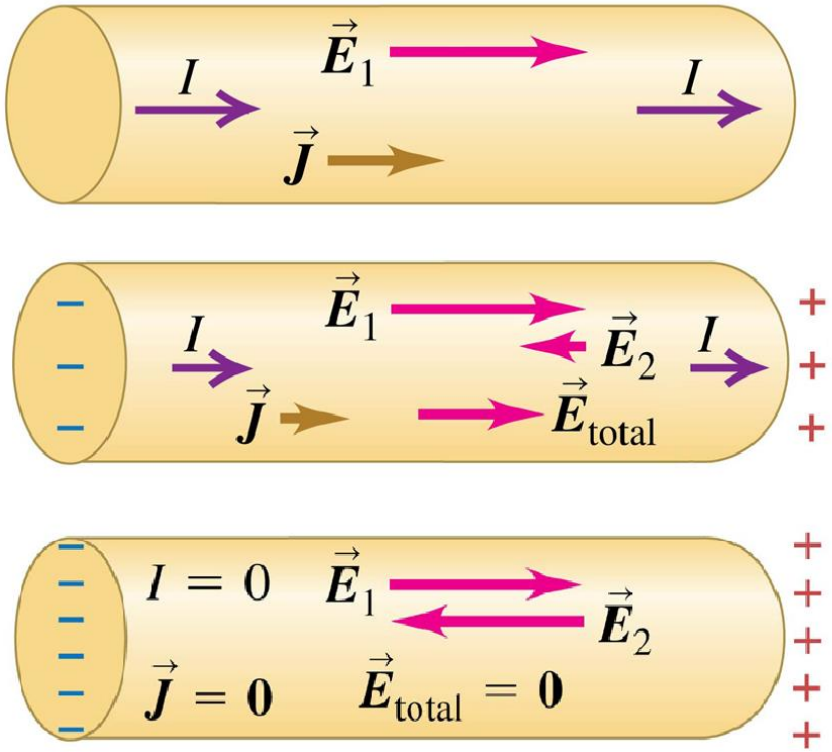
Electromotive force and circuits

- Just like a water fountain needs a pump, an electric circuit needs a source of **electromotive force (emf)** to sustain current.

Electromotive force and circuits

- In order to have a steady current, a circuit must be a **closed loop**.

- If you apply a field $\vec{E}_1$ to an isolated conductor, the field moves charges which creates a second field $\vec{E}_2$ in opposite direction.
  - Eventually you will have $|\vec{E}_1| = |\vec{E}_2|$ and the flow of charges stops.
  - No collective flow of charge means no current.



Electromotive force and circuits

- A **battery** or **power supply** is a source of electromotive force (**emf**, ε) and supplies a potential difference in the circuit to allow charge to flow.
  - **NB**. Emf is not a force, it's energy per charge (unit V).

- A typical flashlight uses two AA batteries in series. Each battery provides 1.5 V of emf.
  - Ie. Each battery does 1.5 J of work on each C of charge.
  - Also means the potential difference between the ends of the AA battery is 1.5 V.

Internal resistance in a circuit

- Real sources of emf contain **internal resistance** r.

- A 12 V power supply has terminal voltage V_{ab} when it is connected to the lightbulb:

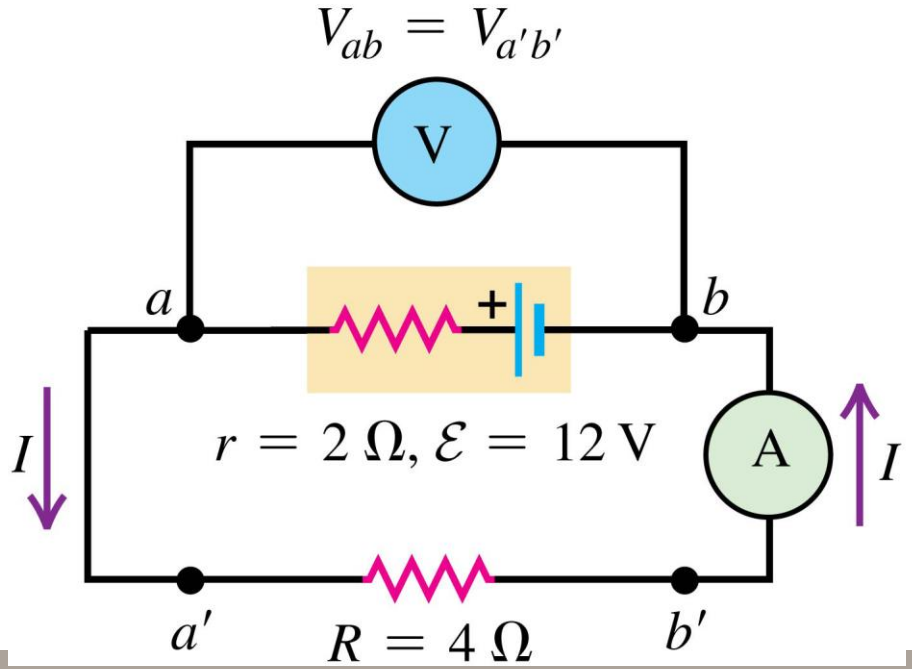
$$V_{ab} = \varepsilon - Ir$$

- Typically, $r \ll R$ in most electronic devices
  - Example in the image:
    R of the lightbulb is ~250 Ω.
    r of the power supply is ~2 Ω.

Ex. 25.5 – A source in a complete circuit

- A battery with emf $\varepsilon = 12\ \mathrm{V}$ and internal resistance $r = 2\ \Omega$ is connected to a resistor $R = 4\ \Omega$ as shown.

  Determine the readings of the voltmeter (measuring V_{ab}) and the ammeter (measuring I).



Solution:

$$V_{ab} = \varepsilon - Ir = V_{a'b'} = IR$$

$$\boxed{I} = \frac{\varepsilon}{r+R} = \frac{12\ \mathrm{V}}{6\ \Omega} = \boxed{2.0\ \mathrm{A}}$$

$$\boxed{V_{ab}} = 12\ \mathrm{V} - (2.0\ \mathrm{A})(2\ \Omega) = \boxed{8.0\ \mathrm{V}}$$

Potential changes around a circuit

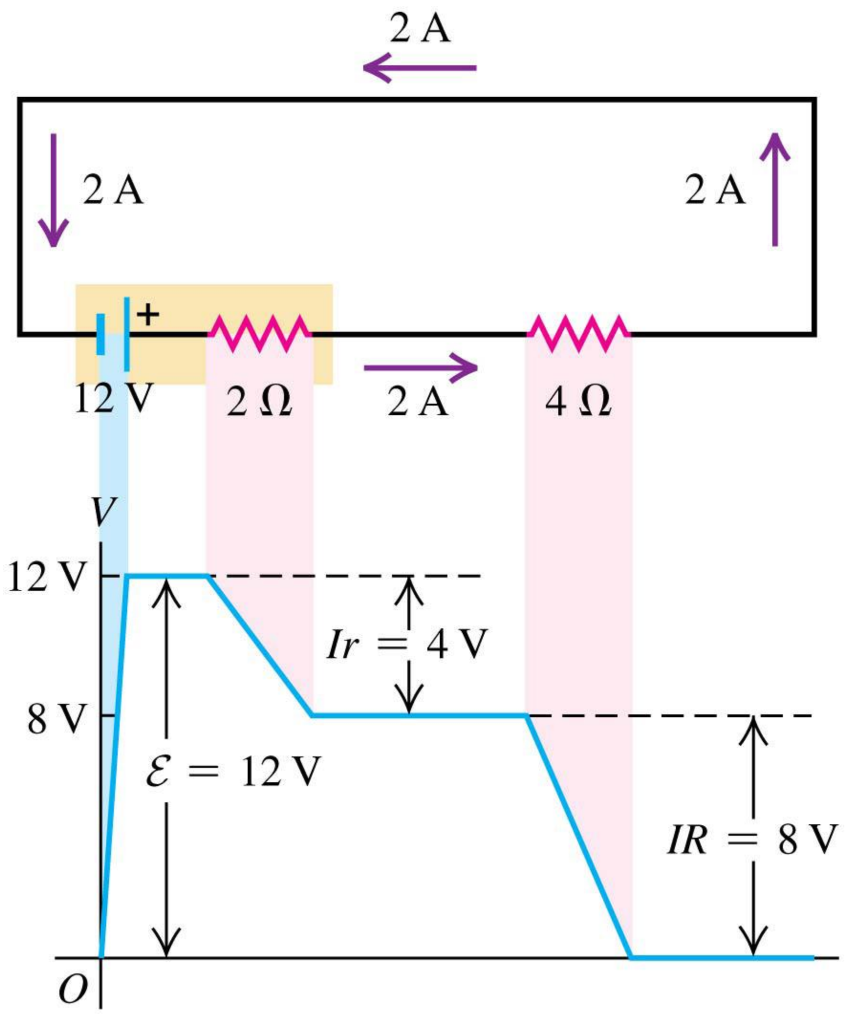
- For the circuit shown: $I = V/R = 12/6 = 2\ \mathrm{A}$

- In a closed circuit, the net change in potential must be equal to zero:

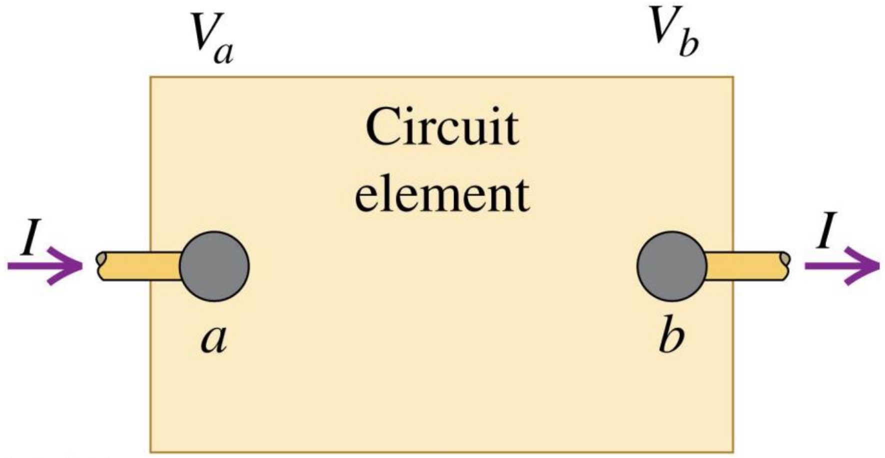
$$\boxed{\Sigma \Delta V} = \varepsilon - Ir - IR = \boxed{0}$$

- Note: the current is the same at all parts in this “circuit” because all elements are in series.

- We will see this net zero potential again in Ch. 26 when we study **Kirchhoff’s rules.**



Energy and power in circuits



- The box represents a circuit element with $V_{ab} = V_a - V_b$ and current I passing through it.

- If $V_b < V_a$, we have energy transfer **in** to the element (like a toaster).
  - If $V_a < V_B$, energy is transferred *out* of the element (like a battery).

- **Power** is the rate of change of energy transfer:

$$\boxed{P = V_{ab}I}$$

  - Measured in **watts**: 1 W = 1 J/s

The units:

$$\boxed{\mathrm{W}} = \mathrm{V} \cdot \mathrm{A} = \frac{\mathrm{J}}{\mathrm{C}} \cdot \frac{\mathrm{C}}{\mathrm{s}} = \boxed{\frac{\mathrm{J}}{\mathrm{s}}}$$

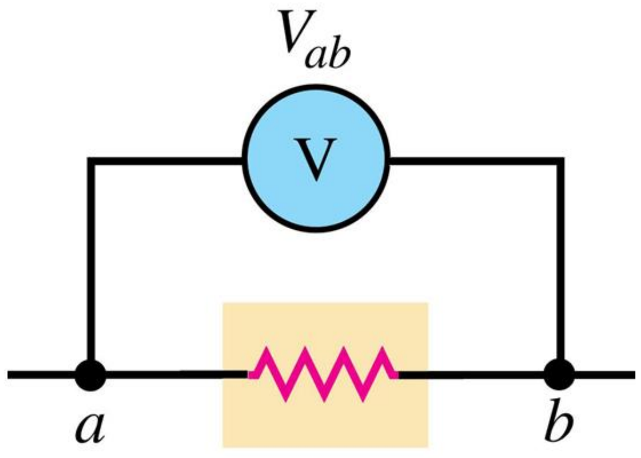
Power and energy to a resistor

- The power ($P = V_{ab}I$) to a resistor ($V_{ab} = IR$) is given by:

$$\boxed{P = I^2R = \frac{V^2}{R}}$$

- The potential is always higher before the resistor (point a) than after (point b) for current moving from a to b.

- The energy transferred into a resistor makes it hot. Some resistor systems (eg. space heaters, toasters) are designed to get hot and due to resistive heating and transfer heat into its surroundings.



Power to/from an emf source

- Power can be output *from* a source (like a battery powering a flashlight):

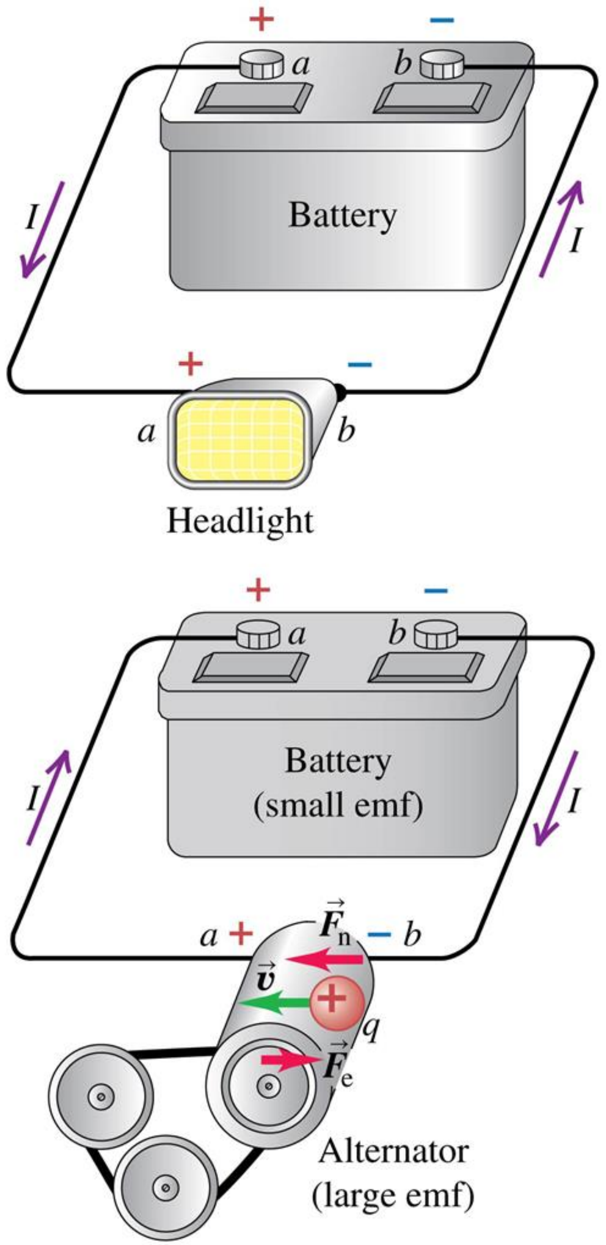
$$\boxed{P} = V_{ab}I = \boxed{(\varepsilon - Ir)I}$$

Work is being done by the source to move the charges through the light.

- Power can also be input *to* a source (alternator charging a car battery):

$$\boxed{P} = V_{ba}I = \boxed{(\varepsilon + Ir)I}$$

Source with larger emf works on and delivered energy to smaller emf.



Exs. 25.8-10 – Power in a circuit

- **25.8**: Find the rate of **energy conversion (energy transfer)** and **energy dissipation** in the circuit below (left figure) for each element as well as the net power output of the battery.
- **25.9**: What if the $4\,\Omega$ resistor was replaced with an $8\,\Omega$ one?
- **25.10**: What if the resistor was removed (right figure)?